A Comparison of Hierarchies for Decentralized Vibration Control

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Abstract

The purpose of this work is to compare the performance of various types of hierarchical organizations for the decentralized control of vibrations in a beam. A decentralized control system is one that consists of numerous localized controllers acting on a single plant, in order to achieve a global control objective. Each of these localized compensators possesses its own computational capability, sensor, actuator and the ability to communicate with neighboring controllers. The primary focus of this work is to compare the performance of various hierarchical organizations of the controllers. The ability of such control systems to perform nearly as well as centralized compensators is demonstrated. The capability of hierarchies to act as decentralized modal observers is also discussed.

Introduction

Although the application of active control to vibrations has been investigated from many years, the extension of this technology to large-scale systems has been thwarted, in part, by an overwhelming need for computational effort, data transmission and electrical power. This need has been overwhelming in the sense that the potential applications (particularly aerospace systems) are unable to bear the power, weight and complex communications requirement of large-scale centralized control systems. Recent developments in MEMS devices and networked embedded technologies have changed the focus of such applications from centralized control architectures to decentralized ones.

A decentralized control system is one that consists of many autonomous, or semi-autonomous, localized controllers called nodes, acting on a single plant, in order to achieve a global control objective. Each of these nodes has the following assets and limitations: 1) a

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computational processor with limited computational capability and limited memory, 2) oversight of a suite of sensors, actuators and the necessary signal conditioning hardware and 3) a network communications link (either wired or wireless) with neighboring or regional nodes and with limited bandwidth. The objective of a decentralized controller is the same as for a centralized control system: to maintain some desirable global system behavior in the presences of disturbances. However, decentralized controllers do so with each node possessing only a limited amount of information on the global systems response. The information available to each node is limited to the local sensor signal, actuator signal and any information that is shared among nodes over the network. Exactly what information is shared among nodes, and how that information is used, is the topic of this investigation.

The field of decentralized control has been the topic of numerous investigations for over 30 years [1]. Most of these studies have considered "weakly connected" systems or architectures wherein each node only experiences a few of the degrees of freedom of the entire system while being weakly connected to other parts of the system. Robotic swarms are a good example of weakly connected systems and have been the topic of many research projects in recent years [2,3,4]. Decentralized control has been considered in a few vibration control projects for application in space structures [5,6,7]. Hierarchical decentralized approaches have also been considered for control of buckling in beams as well [8]. The work reported in this manuscript was inspired by, and is and extension of the work in References 5,6 and 8. It builds on the work of How et al by considering multi-layer hierarchies [5,6]. The current work also extends the work of Hogg and Huberman by employing hierarchical control of continuous dynamic systems. Another unique feature of this work is that it considers the decentralized control by localized controllers all of whom experience the complete dynamics of the system to be controlled. That is to say that each node "sees", or obtains sensor information that contains contributions from, all of the global systems states (or degrees of freedom). This is in contrast to decentralized control of robotic swarms wherein each node (robot) only experiences it's own dynamics with weak coupling to its neighbors.

This work specifically addresses the decentralized control of vibration in a simply supported beam. The beam is equipped with numerous sensors and actuators. Decentralized compensators are designed which interact with each other by sharing sensor and actuator information as defined by various hierarchies. The performance of these hierarchical, decentralized control approaches are evaluated by comparing their performance with a centralized control systems that uses the same sensors and actuators and that expends an equal amount of control energy. The discussion begins with a general discussion of hierarchical, decentralized control. This is followed by the development of a specific example; namely a simply supported beam. This includes beam modeling, control design methodology and hierarchical organization. Finally, results are presented which demonstrate the effectiveness of various hierarchies in active vibration control.

Hierarchies for Vibration Control

The thrust of this work is to investigate the effectiveness of various hierarchies for decentralized vibration control. In this context, hierarchies consist of layered abstract entities, called (borrowing terminology from the computer science realm) agents. Each agent receives information from other agents and/or directly from sensors. Behaving like an independent controller, each agent processes it's inputs in a continuous manner to produce command outputs. This output is either passed on to another agent and/or directly to an actuator. The hierarchy is the organizational structure that defines how input/output information is shared among agents. An example of such a hierarchy is shown in Figure 1.

In this hierarchy, the bottom tier of agents (i.e. A_{11}, A_{12}, …, A_{19}) are linked directly to the sensors and actuators that provide for feedback control. In general, these lowest tier agents may command a heterogeneous suite of sensors and actuators. A practical application for such a hierarchy would consist of numerous nodes distributed over the system to be controlled. Each node would host one or more agents and support the input/output needs of these agents. The hierarchical communications can be described as either horizontal or vertical. Horizontal communications are between agents at the same level while vertical communications are between agents of higher/lower levels. Lowest level agents will deal directly with sensor and actuator signals while higher-level agents will typically deal with "digested" or combined versions of these signals.

In general, agents for vibration control, and the hierarchies in which they reside, can be of any form. Agents may be any type of compensator such as constant gain, dynamic, adaptive or nonlinear. The hierarchies may also be of any form one can imagine. However, in order to develop an agent that can be hosted by processors of limited computational ability, only simplified hierarchies and agents will be considered here. These hierarchies will be limited to purely horizontal or purely vertical ones. Furthermore, the hierarchies will be regular in that each agent of a layer has similar connectivity. Lastly, the considered here are limited to constant gain output feedback regulators. Brief descriptions of the two types of hierarchies follow.

Horizontal Hierarchies

A purely horizontal hierarchy is one that consists of only a single layer of agents that communicate with each other as shown in Figure 2. Although Figure 2 shows communications links only with immediate neighbors, agents may communicate directly with any other agent. In general the information that is exchanged may be any form of sensor and actuator signals (including filtered versions of the signals) or a combination of both. However in this investigation, the information that is exchanged among agents is limited to unfiltered sensor data. The degree of information sharing is referred to as the "reach" of an agent. An agent who has a reach of R will have access to 2R+1 sensors; R sensors to the left, R sensors to the right and its own sensor. Agents located near the edge of the domain will simply have non-existent sensors omitted. Each agent then creates a single output based on the available sensor signals. This output is used to command the actuator associated with that agent.

Vertical Hierarchies

A purely vertical hierarch is one that consists of numerous layers of agents that communicate only with agents above or below them in the hierarchy (no communication with agents in the same layer). An example of a vertical hierarchy is shown in Figure 3. In general, the information, which is exchanged, may be filtered sensor data, filtered control signal data, or a combination of both. In this investigation the shared information is limited such that any agent not in the lowest level will receive the instantaneous average of all sensors below it in the hierarchy. The lowest level agents simply receive the single sensor signal as inputs. A single output is produced by each agent and is sent to all agents below it in the hierarchy. Command signals received by each agent from higher-level agents are added to the command signal produced by that agent. This command is then passed down to all lower level agents. At the lowest hierarchical level, the command signal sent to the actuator consists of the sum of all command signals coming from agents above the actuator in the hierarchy.

Two parameters of the vertical hierarchies are considered here. First is the depth of the hierarchy (i.e. the number of layers) and the second is the relative performance penalty used in each layer. When each agent is designed a performance penalty is associated with the design (see below). Thus, the design of each agent may be performed with different penalties on the system performance. The use of different performance penalties is addressed here, but is limited such that all agents on the same layer have the same penalty. Agents on different layers may have different penalties. This approach allows for some interesting properties in the control performance as will be demonstrated.

Hierarchy Control System: Modeling and Design

The objective of this work is to investigate the effectiveness of decentralized control as applied to the reduction of vibration in a distributed parameter system. To do so the control of a simply supported beam is considered. The objective of the decentralized controller is to minimize the beam response to a random disturbance input by applying various hierarchical control architectures through collocated point force/point velocity feedback. The beam model and decentralized controller design are described followed by a discussion of the results obtained with various hierarchies.

Plate Dynamics

The plant under consideration is a simply supported beam subject to a point force disturbance and to point force actuation collocated with point velocity sensing. The beam is modeled using Galerkin's technique to discretize the linear equations of motion. The partial differential equation of motion is

$$0 = EI\nabla^4 w(x,t) + \rho h \frac{\partial^2 w(x,t)}{\partial t^2} + f_d(x,t) + \sum_{k=1}^{K} f_c(x,t) \tag{1}$$

where *w(x,t), E, I,* ρ and *h* are the beam displacement, modulus of elasticity, density and thickness respectively. The beam is acted upon by a disturbance force, f_d, and K control forces, f_c. A separable solution is assumed using the *in vacuuo* beam eigenfunctions and generalized coordinates of the form

$$w(x,t) = \sum_{n=1}^{N} \psi_n(x) q_n(t) = \sum_{n=1}^{N} \sin\left(\tfrac{n\pi x}{L}\right) q_n(t) \tag{2}$$

where, $\Psi_n = \sin(n\pi x/L)$ are the mode shapes and $q_n(t)$ are the generalized coordinates. Substituting Eq. (2) into Eq. (1), multiplying by an arbitrary expansion function, $\Psi_m(x,y)$, and integrating over the domain yields a set of ordinary differential equations of the form:

$$0 = M_n \ddot{q}_n(t) + K_n q_n(t) + \sum_{k=1}^{K} Q_{kn}^{c}(t) + Q_n^{d}(t) \tag{3}$$

where M_n and K_n are the modal mass and stiffness and Q^c_{kn} and Q^d_n are the control generalized forces and the disturbance generalized forces. The beam model can be cast in state variable form as follows [9]:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx} \end{aligned} \tag{4}$$

where **x** is the state vector containing the generalized coordinates, $q_n(t)$ and their derivatives, **u** is a vector of control and disturbance forces, and **y** is the beam vibrational velocity at each control point.

Agent Design: Output Feedback for Decentralized Control

In order to fit within the constraints of the envisioned system the hierarchies considered in this investigation are composed of relatively simple agents. Each of these agents is an optimally designed output feedback compensator. These compensators may will either be single-input/single-output (SISO) or multi-input/single-output (MISO) depending on the type of hierarchy in which they reside. As described previously, horizontal hierarchies will employ MISO agents and vertical hierarchies will employ SISO agents. Those agents that occupy the lowest hierarchical levels receive one or more sensor signals as inputs. Agents occupying higher levels of vertical hierarchies receive the average of all point velocity measurements coming from sensors below them in the hierarchy. Thus, all agents above the lowest level are single input compensators. The lowest level agents may have one, or multiple inputs depending on whether they are part of a horizontal hierarchy or not.

All agents are optimal, constant gain, output feedback compensators and all are designed in the same manner. Since the outputs considered hear are from point velocity sensors (or averages of point velocity measurements), the output feedback control amounts to rate feedback which has desirable stability and robustness properties [10]. In this case, the system output consists of point velocity sensors and the inputs are point force actuators. Therefore, the control forces are based on system output such that

$$\mathbf{u_c} = -\mathbf{Ky} \tag{5}$$

where $\mathbf{u}_c$ is that portion of the system input corresponding to the control forces, **K** is the feedback gain matrix and **y** is the vector of system outputs containing point velocity measurements at each node. The feedback gain matrix can be found by minimizing the cost functional [11]

$$J = \int_{t_0}^{t_\infty} \left[\mathbf{x}^{\mathrm{T}} \mathbf{Q} \mathbf{x} + \mathbf{u}_{\mathbf{c}}^{\mathrm{T}} \mathbf{R} \mathbf{u}_{\mathbf{c}} \right] dt \tag{6}$$

where **x** is the system state vector, **Q** is a semi-positive definite performance weighting and **R** is a positive definite control effort penalty. Details concerning the calculation of a feedback gain matrix which minimizes equation (6) can be found in Reference 11. The weighting matrix **R** was set equal to 1 in all cases (a scalar since all agents produce one output). The performance weighting matrix, **Q**, was set equal to the identity matrix of appropriate dimension multiplied by a scalar whose magnitude will be discussed shortly.

$$\mathbf{Q} = \alpha \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \tag{7}$$

Since the outputs of the system are point velocities, the objective of the control system is to minimize the mean-square vibration response at each node while maintaining a suitable control effort. Since numerous nodes will be considered, the sum of the squares of the point velocities represents an approximation of the total structural kinetic energy.

Each agent was designed independently based on the open loop plant and employing the method outlined previously. Once each agent was designed, all agents were appropriately connected to the open loop plant. A new system arrangement was constructed which had the closed loop system control signals as outputs and the disturbance as the only input. The H_2-norm of this system was calculated. If this norm was not equal to one, then the scalar multiple of the output weighting matrix, α, was adjusted. Then all agents were redesigned and the process was repeated. This iteration was continued until an acceptable accuracy was achieved. There were two reasons for iteratively calculating the agents in this manner. The first reason was to ensure a fair comparison basis for different hierarchies. The quantity being preserved among all systems is the H_2-norm between disturbance input and control signal output. This quantity is proportional to the total energy contained in all control signals [9]. Therefore, if all hierarchies have the same H_2-norm then they will expend an equal amount of control energy. In the specific case of velocity feedback, since the loop is closed between velocity and force, this also implies that the total control power is equal.

The second reason for iteratively solving for each agent was to ensure a control system design that one could reasonably expect to achieve in an experimental setting. If the $\mathcal{H}_2$-norm between disturbance and control is set equal to one, this means that the total energy expended by the control system is equal to the disturbance energy [9]. Furthermore, this level of control effort has been found to be a reasonably achievable goal, if somewhat optimistic, for experimental implementations of control systems [12].

Comparison of Hierarchy Performance

In this section, the ability of various hierarchies to control the vibration of a simply supported beam is compared. In all of the following cases, the plant to be controlled consists of a simply supported beam with 32 collocated sensors and actuators. These are equally spaced along the length of the beam. The metric used to evaluate system performance is the singular value plot of the system frequency response between the disturbance input and all sensor outputs. The two norm of the system between the disturbance input and all sensor outputs is also used as an overall metric. Furthermore, as a basis of comparison, the performance of a centralized compensator will be shown. This compensator consists of an optimally designed output feedback regulator that utilizes all sensors and actuators while expending the same amount of total energy as the decentralized compensators.

Horizontal Hierarchal Control

The first to be considered is the horizontal hierarchy. These hierarchies consist of a single layer of agents that communicate with each other to varying degrees. The primary variable considered here is the reach of each agent; or the number of neighboring sensor signals that each node employs to affect its control. A comparison of the performance of various hierarchies is shown in Figure 4. This plot shows the singular value spectrum for the open loop system, several closed loop horizontal hierarchies, and a closed loop centralized controller. Figure 4 demonstrates that the performance of a decentralized horizontal hierarchy improves as the reach of each node increases. Furthermore, when the agent reach is 5 the performance of the system cannot be visually differentiated from the performance of the centralized controller. This is clearer in Figure 5 which is an expanded view of Figure 4. Note that the curves for a centralized controller and a horizontal hierarchy with reach of 5 are virtually indistinguishable. The horizontal hierarchy with a reach of 10 significantly outperforms the centralized controller near the lowest system modes. However, over the entire frequency range the centralized controller does achieve the best performance. This conclusion is demonstrated in Table 1 which shows the two-norm between disturbance and sensors for each of the systems.

Table 1 Comparison of $\mathcal{H}_2$ norm for closed loop performance.

SYSTEM TYPE	$\mathcal{H}_2$ NORM
Open Loop	8.24
Horizontal Hierarchy, Reach = 1	3.35
Horizontal Hierarchy, Reach = 2	3.22
Horizontal Hierarchy, Reach = 5	3.19
Horizontal Hierarchy, Reach = 10	3.26
Centralized Control	3.19

Note that while decentralized control performance approaches that of the centralized compensator, the centralized compensator either outperforms in the low frequency regime (which is the dominate contributor to the overall system response), outperforms in the total attenuation, or both. However, the fact that the decentralized approach can come very close to the centralized performance is very promising. A decentralized control system can nearly equal the performance of a centralized controller simply by sharing sensor data with neighboring nodes.

Vertical Hierarchal Control

In vertically hierarchical organizations there are two parameters that are considered. First is the depth of the hierarchy (i.e. the number of layers) and the second is the relative performance penalty used in each layer. Three specific vertical hierarchies are considered here as shown in Figure 6. These consist of either 2, 4 or 6 layers. Recall that each agent in the upper layers receives the sensor data from all agents below it in the hierarchy. These sensor signals are averaged to produce the input to that compensator. A constant gain is applied to the sensor signal average to create a control signal. This control signal is sent to all agents lower in the hierarchy, which is added to their own control signal.

A comparison of the performance of hierarchies with various numbers of layers is shown if Figure 7. In this case, the same performance penalty weight, α, was used to design all agents in the hierarchy. Note that for the lowest natural frequency, all of the hierarchies were able to outperform a centralized controller expending equal energy. This is more visible in Figure 8 which shows an expanded view of the performance comparison. As can be seen, the centralized controller performs better for all modes above the first mode. It is clear when one compares the performance of vertical and horizontal hierarchies that the horizontal hierarchy is better suited to the task of minimizing the overall beam response. However, vertical hierarchies have some interesting capabilities as discussed below.

A topic of some interest in centralized control has been the ability to target specific modes for attenuation. This presents an interesting problem in decentralized control because of the need to observe the system globally in order to identify modal contributions. Vertical hierarchies are capable of addressing this problem by acting as virtual sensors with selective modal sensitivity. Consider the 4-layer hierarchy of Figure 6. At the highest level of the hierarchy, the agent receives the average of all sensors as the input. This average will be zero for all even modes on the beam since their mode shapes are integer multiple of a full sine wave. So, by increasing the relative performance penalty when designing this agent, one can tune the hierarchy to target the odd modes for attenuation. This modal selectivity is demonstrated in Figure 9 which shows the transfer function of one agent from each layer of the 4-layer hierarchy. Note that the layer 1 agent (the bottom layer) is sensitive to all modes while layer 4 (the top layer) is only sensitive to the odd modes. Furthermore, each layer has a maximum frequency of sensitivity which is related to modal wavelength relative to the sensor spacing. Thus, vertical hierarchies can be used to generate specific modal sensitivities in a decentralized manner. Such a capability would be useful, for example, in active structural acoustic control where the odd modes are the most efficient radiators [13].

In order to take advantage of this ability to target specific modes for attenuation each layer must be designed so as to weight the effort toward the layer that targets the modes of interest. Results of such an application are shown in Figure 10. In this case, four different 4-layer hierarchies were designed. In each case the cost weight, α, of one of the layers was set 10 times larger than the other layers. Therefore, in each of the four cases a particular layer exerts more control effort than the other layers. This allows the vertical hierarchy to target specific sets of modes. Relating back to our previous example, when the highest-level layer receives the largest weight penalty the control system targets the odd modes for attenuation as demonstrated in Figure 10. Note that in this case the first and third modes attenuated considerably while the second mode is only attenuated a small amount. However, when the layer 1 agent design is weighted more heavily, the attenuation of individual modes is proportional to the sensitivity of layer 1 as shown in Figure 9.

The data of Figures 9 and 10 demonstrate the advantage of employing vertical hierarchies in decentralized control. That is, their ability to perform as virtual sensors in order to observe global phenomena based on processing. This example is employs very simple agents within the hierarchy. More complex agent designs incorporating specific frequency sensitivities for example, could be designed to be even more selective in their modal sensitivity.

Conclusions

The organization, design and implementation of hierarchical, decentralized vibration control has been presented. A general discussion of hierarchies for control was provided along with the advantages of such an approach. Several examples were presented centered on the control of a simply supported beam. It was demonstrated that horizontal hierarchical control is capable of performing nearly as well as a centralized control system. Furthermore, the ability of vertical hierarchies to act as modal observers in a decentralized environment was demonstrated. Such approaches will permit the development of decentralized control systems capable of attenuating specific modes in a structural response.

Acknowledgements

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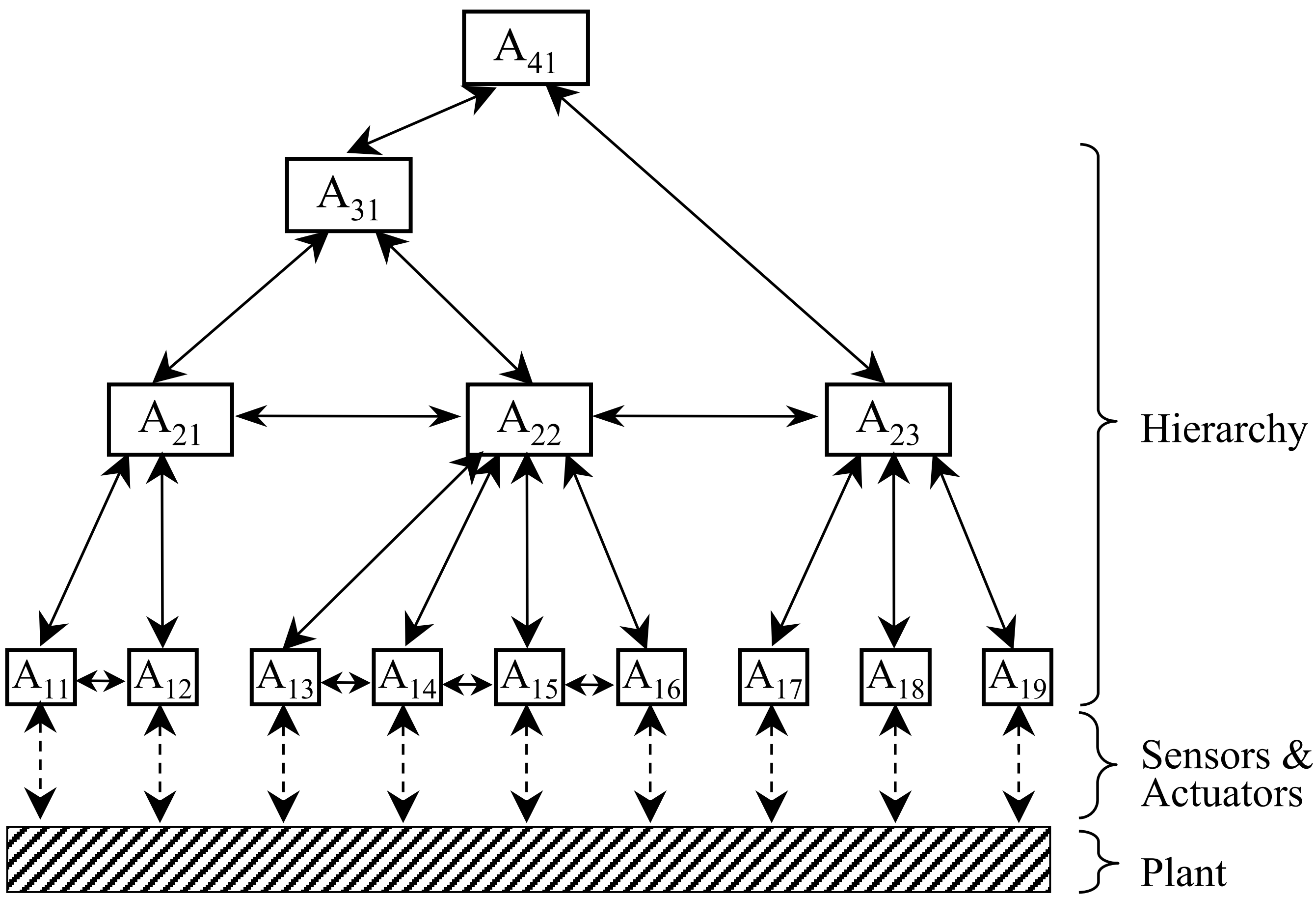

Figure 1 Schematic of hierarchical, decentralized control system.

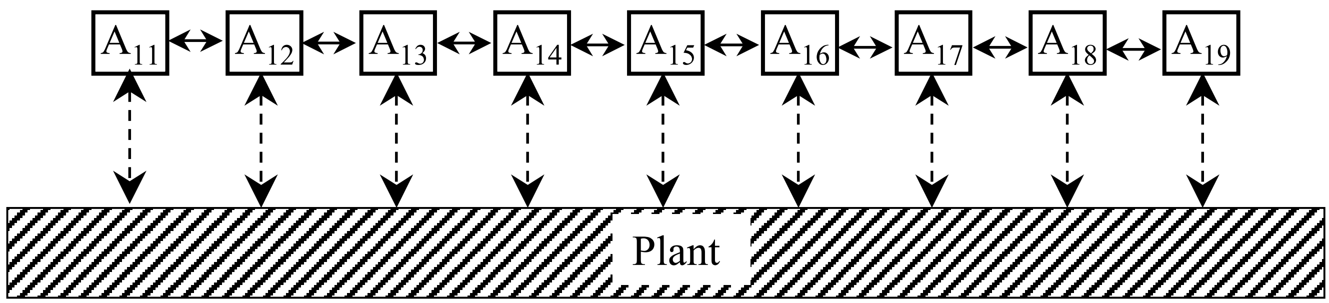

Figure 2 Schematic of horizontal hierarchical control system.

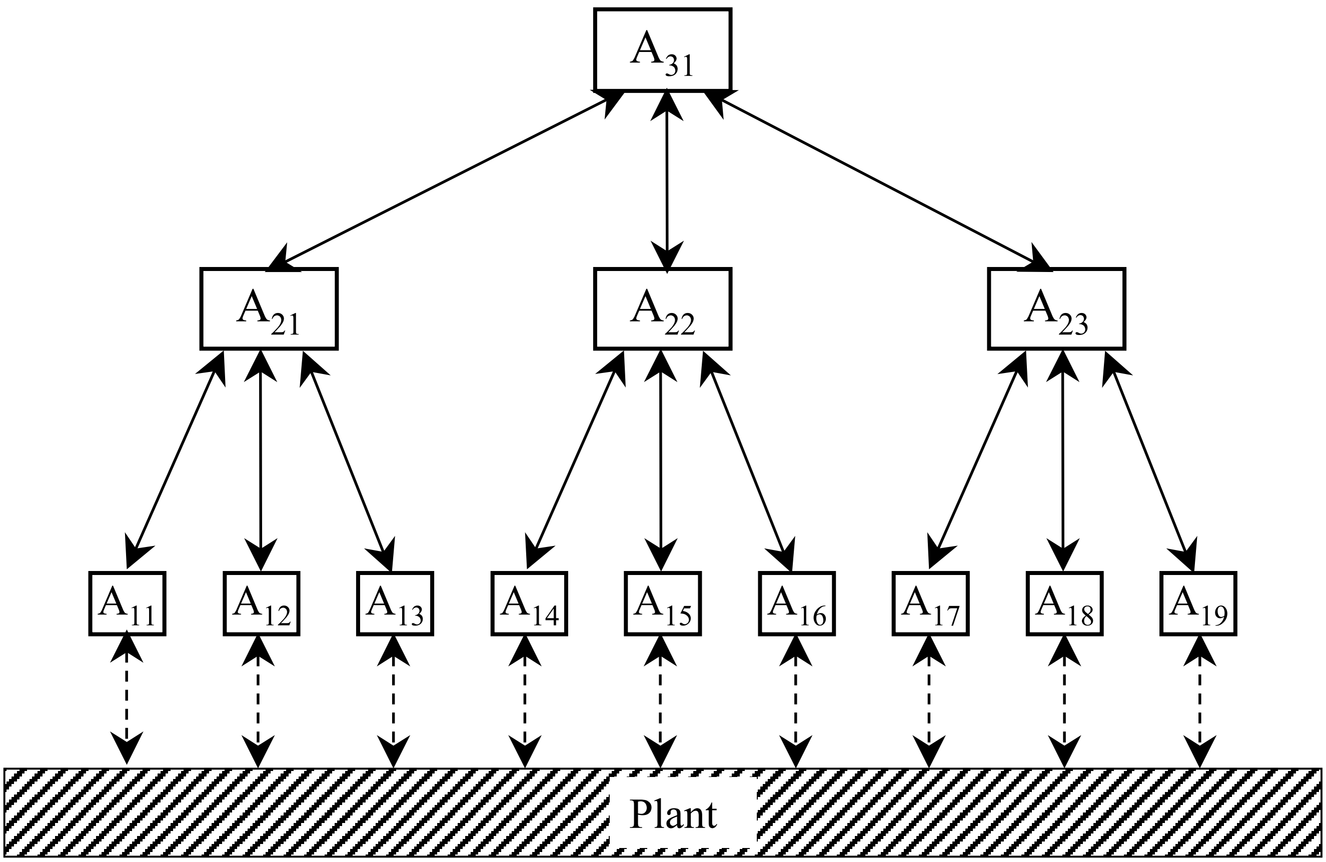

Figure 3 Schematic of vertical hierarchical control system.

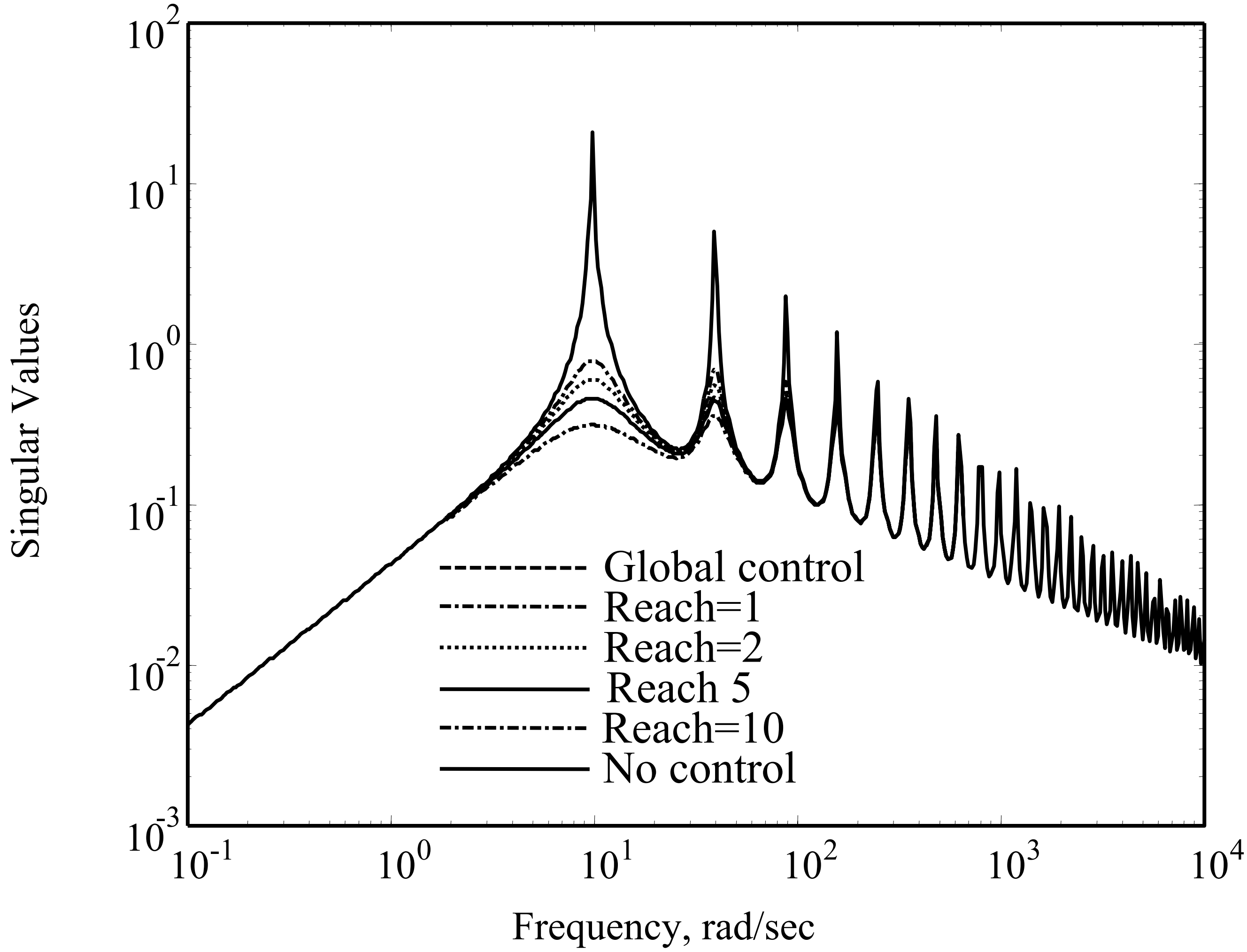

Figure 4 Comparison of closed loop performance between a centralized, global controller and several horizontal hierarchic controllers of varying reach.

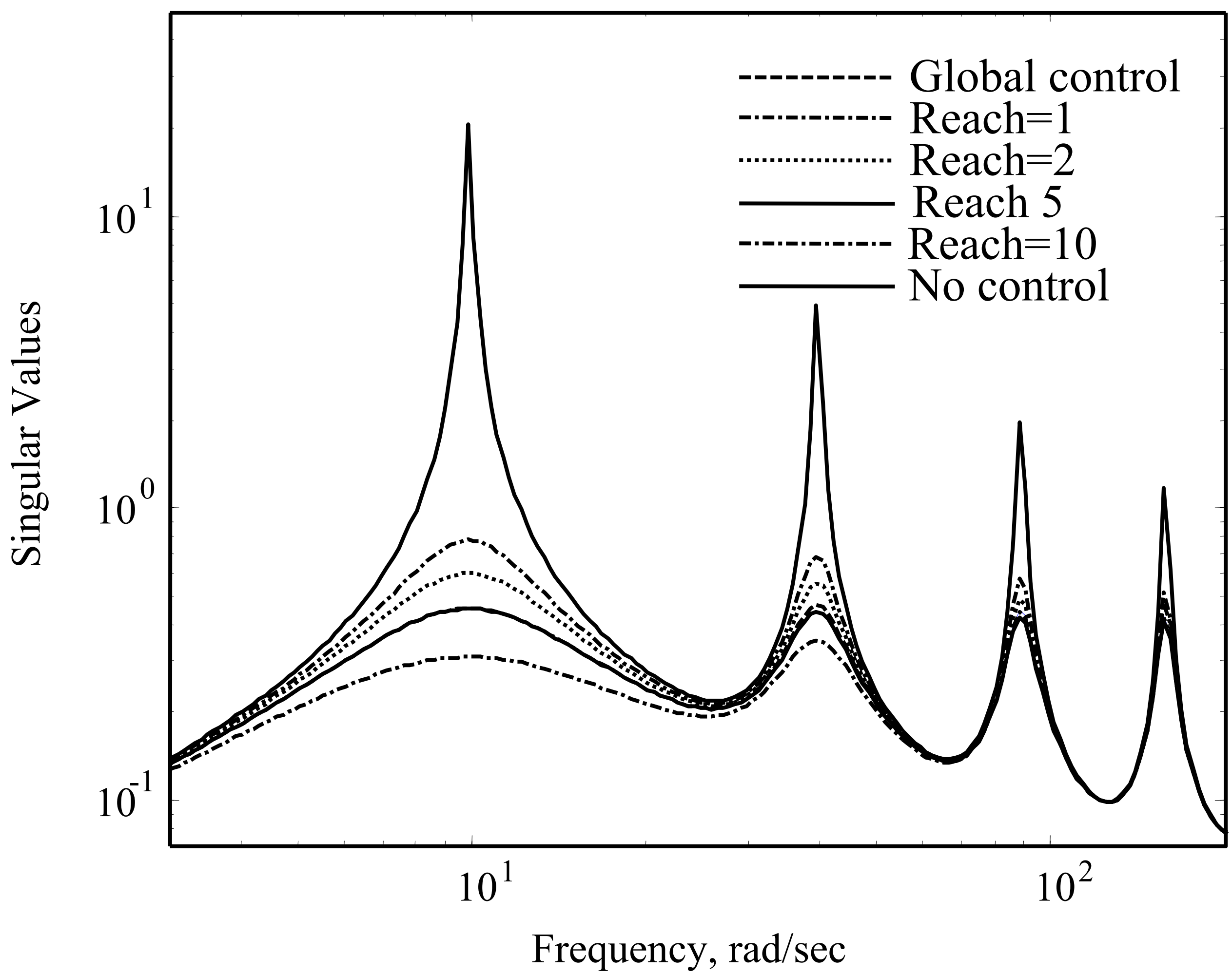

Figure 5 Zoomed view comparison of closed loop performance between a centralized, global controller and several horizontal hierarchic controlers of varying reach.

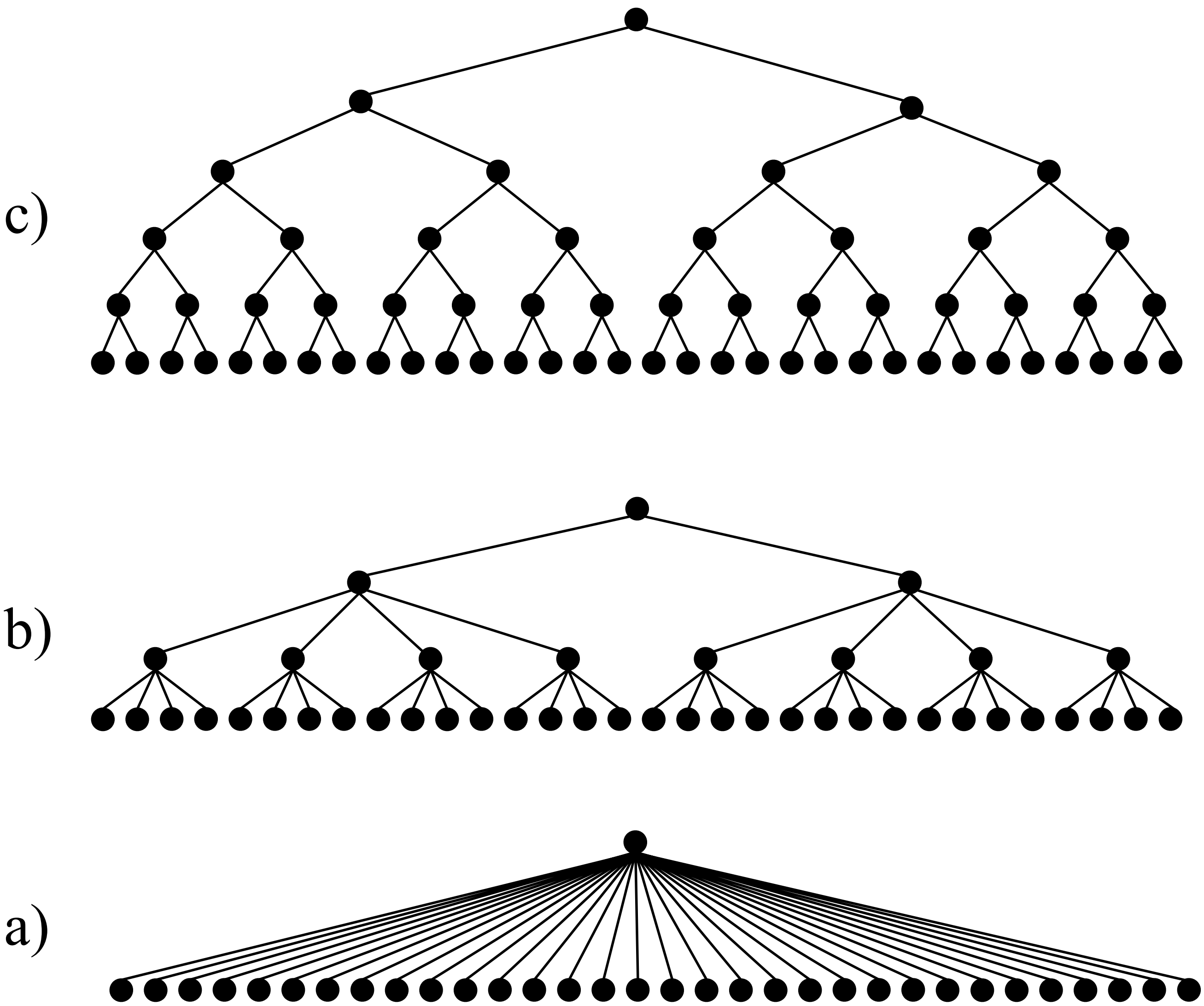

Figure 6 Schematic of the vertical hierarchies used in this study: a) 2-layer, b) 4-layer and c) 6-layer.

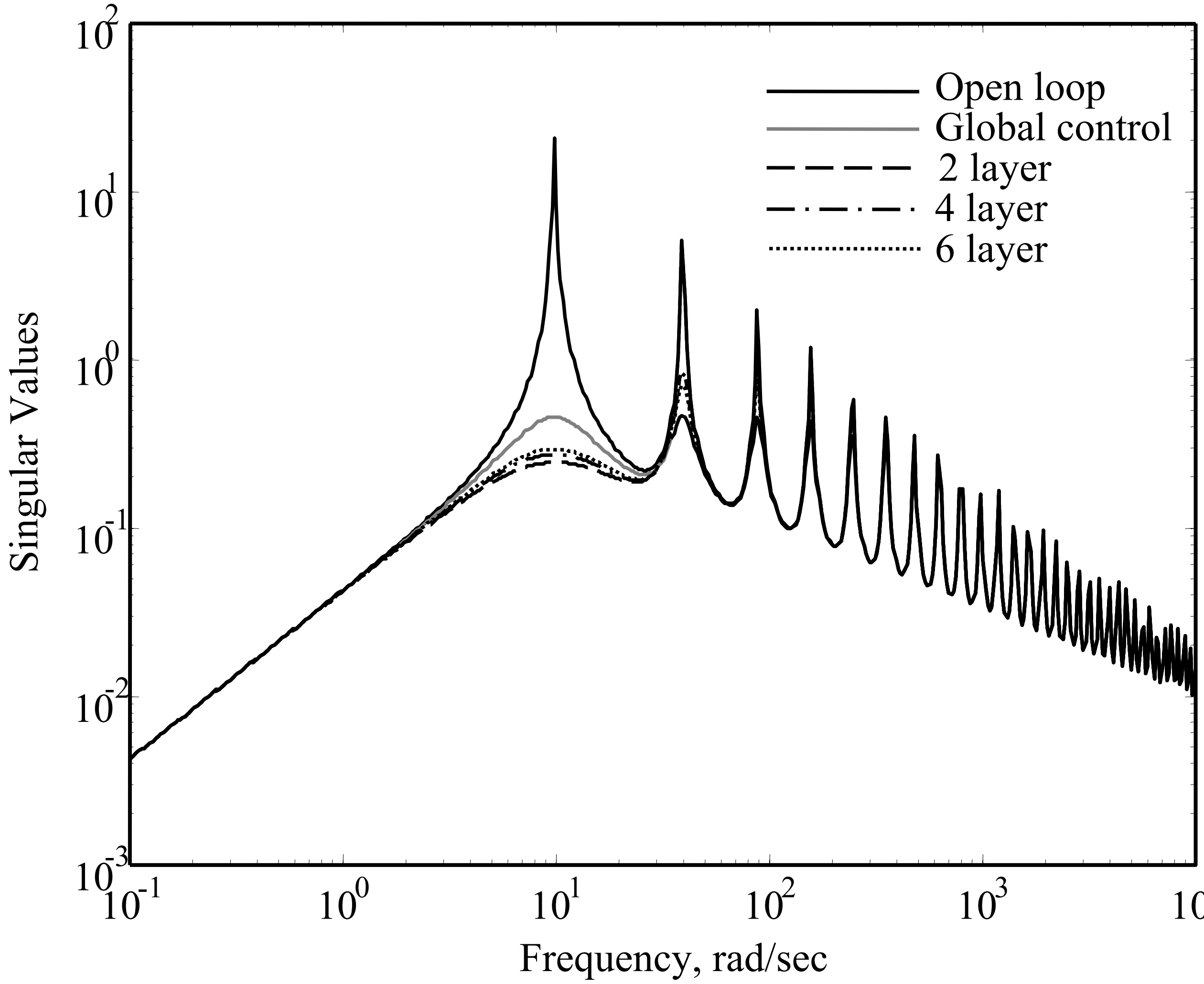

Figure 7 Comparison of closed loop performance between a centralized, global controller and several vertical hierarchic controllers.

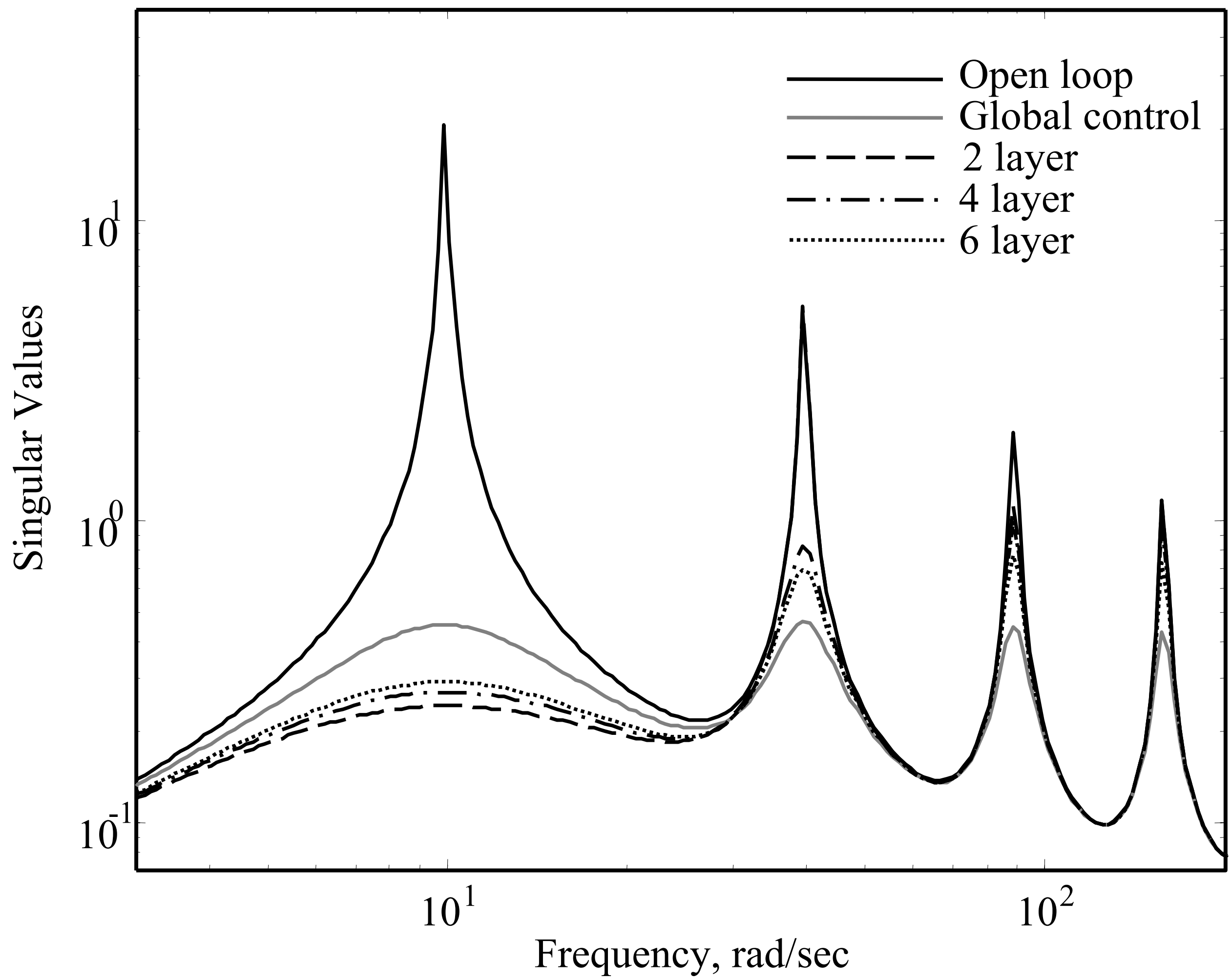

Figure 8 Zoomed view comparison of closed loop performance between a centralized, global controller and several vertical hierarchic controllers.

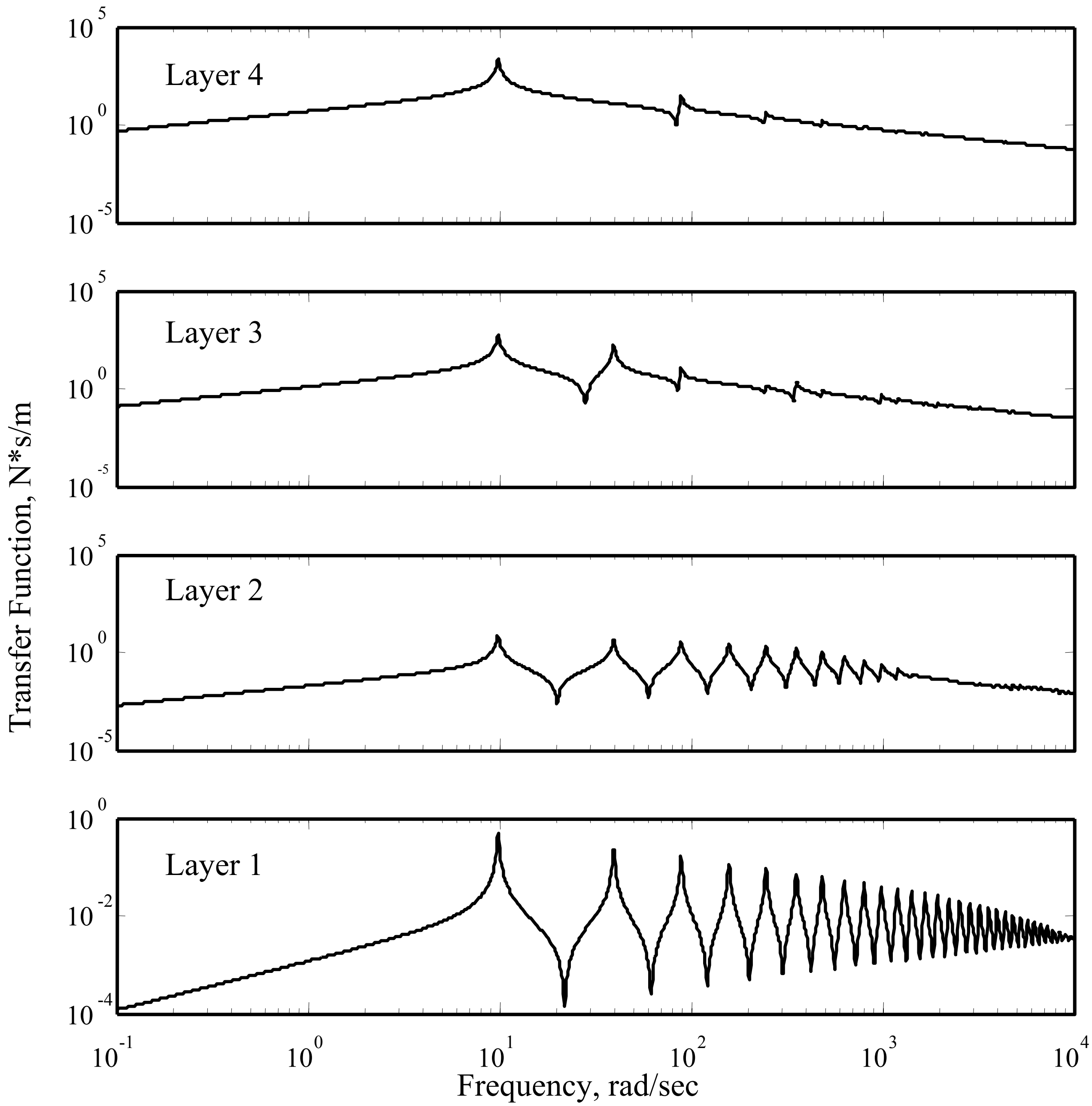

Figure 9 The transfer functions of individual agents situated in specific layers of the 4-layer hierarchy (layer 1 is the bottom).

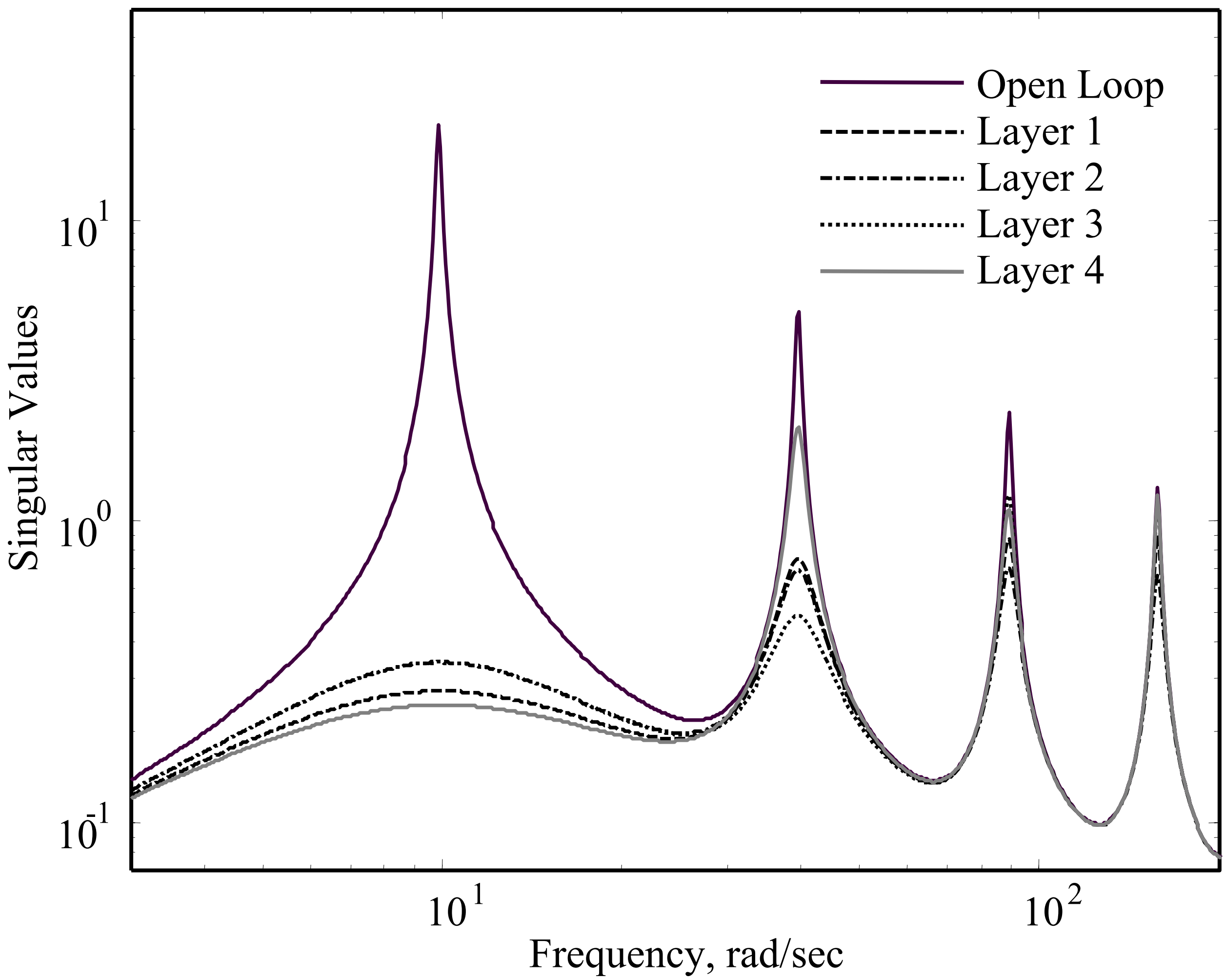

Figure 10 Zoomed view comparison performance for the 4-layer hierarchy when individual layers are designed with a cost weight 10 time larger than the other layers.