Transient Detection and Analysis for Diagnosis of Abrupt Faults in Continuous Dynamic Systems

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Abstract

TRANSCEND, *our system for fault detection and isolation of complex dynamic systems, uses a model based approach to predict and analyze transient effects resulting from abrupt faults in the system. Abrupt faults are attributed to discrete and persistent parameter value changes. Fault isolation is performed by matching features extracted from the transients against those predicted by the model. This paper discusses a statistical signal processing approach to transient detection and analysis using a time-frequency representation of the signal. The approach is robust for the detection task and it provides feature values for the initial fault isolation steps.*

1. Introduction

Model-based approaches for fault detection and isolation (FDI) in continuous dynamic systems employ relations imposed by the system configuration and functionality to compute *residuals* that capture the discrepancies between nominal and observed behavior. Residual computation and analysis is non-trivial for complex systems, primarily because of stiffness, convergence, and intractability problems related to their non-linear dynamic behavior.

We have developed a model-based system, named TRANSCEND, that employs qualitative residual analysis for fault isolation [12]. Figure 1 shows the TRANSCEND architecture. Variables $\boldsymbol{u}$, $\boldsymbol{x}$, and $\boldsymbol{y}$, are the input, state, and output vectors of the physical system, respectively. A standard gain matrix observer scheme [2] tracks the residual, $\boldsymbol{r} = \boldsymbol{y} - \hat{\boldsymbol{y}}$ ($\hat{\boldsymbol{y}}$ is the predicted system behavior), to correct for small deviations in the estimated state vector $\hat{\boldsymbol{x}}$.

Qualitative residual analysis uses a symbol generation module that maps numeric residual values to a symbolic form. Fault detection identifies a discrepancy in the residual, and this initiates the fault isolation task that is performed in two phases: *hypothesis generation* and *hypothesis refinement*. Hypothesis generation uses the diagnosis model, m and the symbolic feature data, $\boldsymbol{r_s}$ at the point of detection to generate a set of hypothesized fault candidates, f_h, and to predicted system behavior, p, for each candidate. A fault candidate is a physical system parameter with a qualitative label that indicates an increase or decrease in the parameter value. During hypothesis refinement subsequent feature values extracted from new measurement data are matched against the predictions to derive the refined fault set, f_r. This process continues until a unique fault is identified or the fault set can no longer be reduced. In more recent work we have pointed out the fundamental limitations of the qualitative approach and demonstrated an extension with a simplified numerical parameter estimation scheme to resolve remaining fault hypotheses that can not be resolved using qualitative techniques [10].

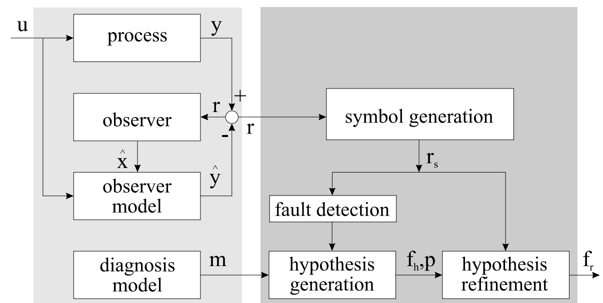


Figure 1. TRANSCEND **architecture.**

In our work, faults are defined as an abrupt (step) change in the value of a model parameter, and this causes transients in the measured signals. Fault detection is the detection of a transient. Consequently, hypothesis generation requires characterization of the transient dynamics predicted by the model, and hypothesis refinement is based on analysis of the evolving transient. The objective is to capture sufficient discriminating features from the transient in a timely manner.

Transient detection and analysis is a challenging problem for realistic, noisy, measurement data, especially when the amplitude of the transient signal is small, resulting in a low signal-to-noise ratio (SNR). The amplitude of the generated transient is directly related to the magnitude of the parameter value change, leading to the intuitive observation that a small parameter change will be more difficult to detect. This paper presents an approach for transient detection and analysis in TRANSCEND that is based on signal detection in the time-frequency (TF) signal representation. The motivation for a time-frequency signal representation is twofold. First, we can design the TF representation so that the transform domain coefficients are sparse, capturing information in an efficient way. Second, the signal representation in the TF domain allows further analysis that can be used in fault isolation.

In the remainder of this paper we first present the principles of predicting fault behavior and qualitative transient detection and analysis in TRANSCEND. We then present the signal representation and detection approach in detail, and illustrate the method with examples.

2. FDI from Transients

The analysis of transients in the TRANSCEND framework is targeted to discriminating among faults, and exploits the behavior predictions generated by the model in constraining this analysis.

2.1. Transients resulting from faults in continuous dynamic systems

The state equation model, for a linear time invariant system is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$
$$y(t) = Cx(t) + Dr(t) \tag{2}$$

In the Laplace domain these equations become:

$$X(s) = (sI - A)^{-1}x(0) + (sI - A)^{-1}BR(s) \tag{3}$$
$$Y(s) = CX(s) + DR(s), \tag{4}$$

where $x(0)$ is value of the initial state vector evaluated at $t = 0$. The time domain solution for any output can be expressed as a sum of complex exponentials, provided all eigenvalues of the system are distinct. A discussion of the link between component parameters and state equations is not part of this paper, but in our work the state equations are easily derived from a bond graph model of the system under consideration [12]. As a result, the terms of the state equation matrices are directly defined in terms of the component parameters.

Consider a system in which an abrupt fault occurs. The change in physical system parameter value results in a change in the system (A,B) and output matrices (C,D), respectively. Changes in parameters of the system matrix are referred to as non-additive, or multiplicative, faults [1]. This class of faults is much more difficult to isolate than additive faults such as sensor and actuator faults. For analysis we set $t = 0$ as the time of failure. The system response for $t > 0$ is then determined by the combined forced response to the input vector $\mathbf{u}$ and the initial value response to a step change in a parameter value that corresponds to the faulty component.

2.2. Representation of transient dynamics

Qualitative analysis of transient signals requires an abstraction scheme to derive qualitative information from the continuous signal dynamics. The notion of signal abstractions was formalized by Milios and Nawab [11]. Signals are viewed in a hierarchy of abstraction levels, where at higher levels, detail is suppressed with respect to lower levels. Higher levels of abstraction support fewer inferences, but they can be tailored to solve specific problems more efficiently. Signal abstractions can be defined in the data domain, a transform domain, or both. As a basis for these signal abstractions, we need a signal model, a description of the signal with respect to an underlying structure [7]. The signal model for the detection problem is the general form of the solution of (4).

In previous work, both the behavior prediction and the transient analysis has been carried out strictly in the time domain [12]. The hypothesis generation algorithm is triggered following the detection of a transient. The direction of its initial deviation, the sign of the transient signal, is required to generate the set of fault candidates. The hypothesis refinement is based on predicting the sign of derivative values. Furthermore it was shown that the detection of discontinuous changes provides important discriminatory information. An interpretation of this method as a Taylor series expansion of the transient immediately after the fault was given in [10]. Robust derivative estimators for this method was discussed in [9].

In this paper we take a different approach to using the model to analyze the transient. We look at the relation between an abrupt parameter value change and the transient dynamics in the Laplace transform domain, and how this can be used to make conclusions about the transient behavior resulting from that parameter change. The dynamic properties of an evolving transient can be directly linked to changes in the parameters of the characteristic polynomial. The relation of the eigenvalues of the system to the component parameters is known and can be used to detect a change in location of the poles of the system. The transfer function also determines whether or not a signal will have an discontinuous change at the onset.

3. Transient Detection and Analysis

We present an approach to detection and analysis of transients generated by faults in continuous dynamic systems, by using a linear time-frequency representation. The detection is then carried out in the transform domain. This leads to robust detection and the extraction of discriminating features used during fault isolation.

3.1. Signal Detection with hypothesis testing

Signal detection is the discovery of a signal in a background of noise. The difficulty of the detection problem is determined by the severity of the noise, and the amount of available knowledge about the signal. Signal detection in time series data is governed by the theory of statistical hypothesis testing. The null, hypothesis, H_0, is the assumption that only noise is observed, and the alternative hypothesis, H_1, corresponds to the presence of a signal in a noisy background. Let $\mathbf{s}$ be the signal of interest and $\mathbf{n}$ is the additive noise. The hypothesis testing is stated as [8]:

$$\begin{aligned} H_0 &: \mathbf{y} = \mathbf{n} \\ H_1 &: \mathbf{y} = \mathbf{s} + \mathbf{n} \end{aligned} \tag{5}$$

We are now interested in constructing a detector that maximizes the probability of detection of $\mathbf{s}$ while ensuring some acceptably small false alarm probability. Such a detector is based on computing the *likelihood ratio* between the alternative hypotheses. When $\mathbf{s}$ is completely known it is possible to build an optimal detector, the matched filter. When knowledge about the signal is limited, a maximum likelihood estimate approach is typically used. The resulting detector is the generalized likelihood ratio test (GLRT). Usually presented in logarithmic form, the GLRT can be expressed as [8]:

$$l(\mathbf{y}) = \max_{\mathbf{s} \in S} \log \left[\frac{p(\mathbf{y}/\mathbf{s}, H_1)}{p(\mathbf{y}/H_0)} \right] \tag{6}$$

The detector with the desired performance is constructed by applying a threshold on l.

The signal detection problem for FDI in realistic measurement data is complex. A transient in a continuous dynamic system is a deterministic signal, for which we have a signal model obtained from (4). However, all parameters of the signal are unknown. The arrival time of the signal is unknown because the time of failure is unknown. Also, the direction and magnitude of the parameter value change that corresponds to the fault are unknown, and, therefore, the amplitude, and phase of the transient are unknown as well.

The challenge now, is to find an appropriate representation of $\mathbf{s}$. This representation should capture the properties of the transient that distinguish it from the background noise. A detector should be designed to exploit as much knowledge of the problem as possible. On the other hand, to avoid unnecessary computational effort, the detector should be limited to recovering only those features of the signal that provide discriminating information among possible faults.

In this paper we propose the use of a linear time-frequency (TF) signal representation to create a non-parametric detector. Transients are non-stationary signals and TF methods are often used to describe non-stationary signals. The term *approximate matched filter* is sometimes used to describe TF based detectors [6]. We note that the derivation of the optimal parametric detector for a pure sinusoidal signal with unknown amplitude, phase, arrival time and frequency results in the computation of the spectrogram, which is a time-frequency representation. The detector compares the peak value of the spectrogram with a threshold [8].

3.2. Signal representation using the Gabor transform domain

Several possible linear TF transforms are available, but we have selected the Gabor transform representation because it uses complex exponential basis functions that appear also in the transient signals of linear systems. For completeness, we recall the definition of the Gabor expansion. For a discrete time sequence $y(k)$ of length L, the finite approximation of the Gabor expansion is defined as [13]:

$$y(k) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} C_{m,n} g_{m,n}(k), \tag{7}$$

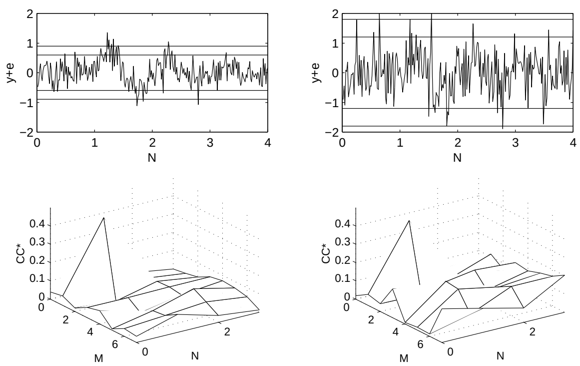
for $k = 0, \ldots, L-1$. $C_{m,n}$ are the Gabor coefficients and N and M are the maximum allowable time and frequency shifts respectively. The synthesis function $g_{m,n}(k)$ is given by:

$$g_{m,n}(k) = \tilde{g}(k - ma) exp(j2\pi nb/L), \tag{8}$$

where the $\tilde{g}(k)$ is the periodic extension of the window function $g(k)$. Some constraints need to be imposed on the parameters of the synthesis function for the transform in (7) to be stable. Details can be found in various treatments of the Gabor transform, e.g. [13].

The suitability of the transform for fault detection in continuous dynamic systems follows from the choice of the window function in (8). Friedlander and Porat suggested a one-sided exponential decaying window function [3], for the purpose of detecting transient signals subject to sudden onset and exponential decay. This matches the fault transients that occur in systems described by (2)-(4). The window function is defined as $g(k) = \sqrt{2\lambda} \exp(-\lambda k) u(k)$, where λ, the damping factor, controls the locality of the analysis, and u is the unit step function. The damped complex exponential basis functions are not orthogonal, but do constitute a frame [3].

Figure 2 illustrates the transform domain representation for a transient signal with two different noise levels. The transform coefficients are plotted in a TF grid with four time bins and 8 frequency bins. The transform is graphically represented as the square of the absolute values of



(a) $\sigma = 0.3$ (b) $\sigma = 0.6$

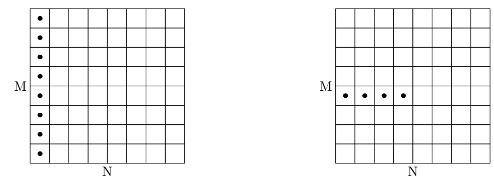
Figure 2. Transient signal with additive Gaussian noise (top) and Gabor transform domain representation (bottom) for different values of the noise standard deviation. Ranges of 2σ and 3σ around 0 are indicated with horizontal markers in the time series data.

the Gabor coefficients, $(CC^{\star})$. Figure 2(b) shows that even when the noise level increases to where visual detection becomes challenging, the signal is clearly recoverable in the transform coefficients. The compactness of the representation in the time-frequency plane is illustrated with this example.

3.3. Fault detection in the transform domain

The next step is to design a GLRT based detector based on the statistical properties of the Gabor transform coefficients. The transform coefficients of white noise are jointly Gaussian [3]. The transform coefficients of a transient signal will have a non-zero mean value in the subset of the time-frequency plane where the signal is located. This becomes the hypothesis H_1. The test is constructed on the probability ratio of non-zero mean valued coefficients for a region of the time-frequency plane. If multiple regions are to be evaluated, multiple tests are required. Each region then corresponds to a different component of the transient signal. Figure 3 illustrates the use of regions in the TF grid.

An important objective in FDI is fast fault detection. That means we must detect the transient as early as possible. It is, therefore, critical that the detector has the ability to analyze the signal from its onset, and draw conclusions from an evolving transient. When a smaller part of the transient signal is available, the amount of signal information in the analysis window is decreased, and effectively, the SNR of the signal is smaller. When the SNR of the signal decreases, the performance degradation affects the partial transient signal more severely. The consequence is that fault detection will require more data in low SNR situations.



(a) TF region representing a single time bin.

(b) TF region representing a single frequency bin in a specific time range.

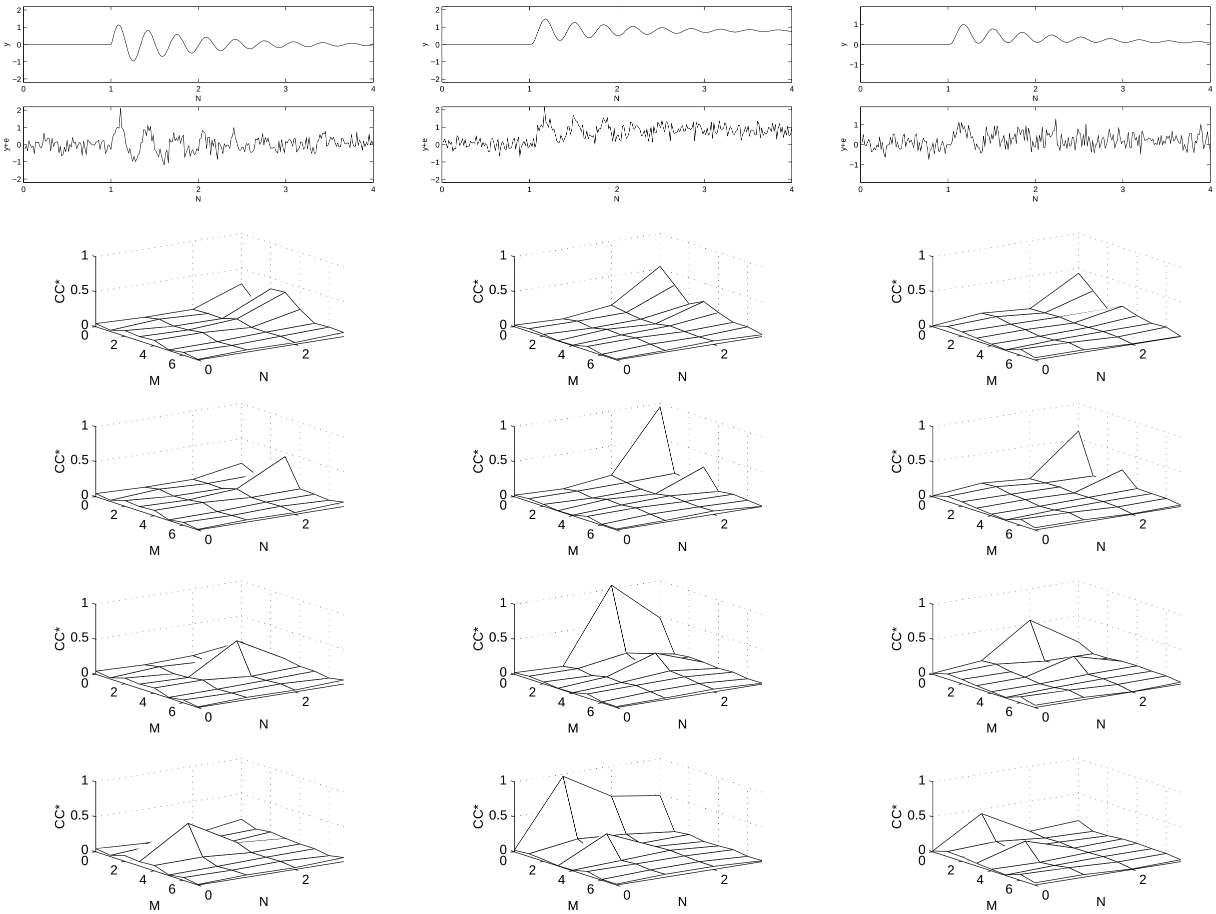
Figure 3. Time-frequency grid with different detection regions. Regions are indicated with dots in the grid cells.

Figure 4 illustrates the transform domain representation for several typical fault transients, as the transient signals progress over time. We indicate the onset of a transient with an *arrival time*, meaning that a smaller arrival time corresponds to having a larger amount of signal available in the analysis window. In the time domain plots, the signal is always shown with the smallest arrival time. Figure 4(a) shows a transient that consists of a single damped oscillating component. This is representative of a parameter change in an energy storage element. The TF domain shows that initially the detector cannot resolve the frequency because not enough signal is available. In following snapshots a damped frequency component emerges, that subsequently vanishes as the signal evolves. The signal in Figure 4(b) has an additional step change, which corresponds to a change in an dissipative component. The step change component is persistent as the signal evolves. Finally, Figure 4(c) shows a transient generated from a fourth order system that also includes an initial zero frequency component that vanishes as the signal evolves.

3.4. Fault isolation in the transform domain

As was seen in Section 2, fault isolation is initiated by hypothesis generation step. Hypothesis generation requires the initial deviation of the signal to generate the initial set of fault candidates using the model. This requires determining the direction of change in the signal (i.e., its sign), which can be derived in a straightforward way from the transform coefficients.

Another important feature for discriminating among fault hypotheses, is to determine if an abrupt change in a parameter value causes a discontinuous change in a measurement signal. A discontinuous change implies that there is no integrating function (associated with an energy storage component) between the faulty parameter and the measured signal. This is recovered from the transform coefficients by evaluating the relative size of the real and imaginary parts. Figure 5 shows two transient signals that differ only in phase, and produce identical detection results.



(a) damped complex exponential signal.

(b) Signal as in (a) with added step change.

(c) Signal as in (b) with an additional damping factor.

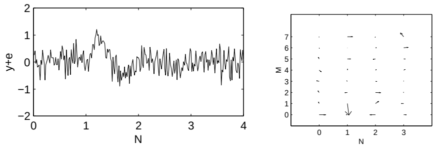
Figure 4. Three transient response signals and their Gabor domain coefficients at various arrival times. Top graph shows signal, and signal with additive Gaussian noise (σ=0.3). Bottom shows coefficients with arrival times [3.5, 3, 2, 1].

However, evaluating the coefficients in polar coordinates reveals the differences between the signals.

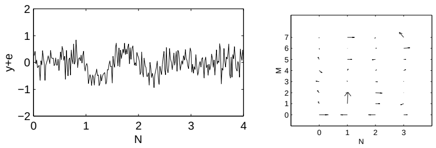
Another feature, the steady state operation of the faulty system, helps discriminate between hypothesized faults in energy storage elements, and fault in dissipative elements. An abrupt fault in an energy storage element results in transient behavior that returns to the previous steady state, however, a fault in a dissipative element results in a change in the steady state behavior. It was pointed out in [12] that the detection of steady state in a signal is quite difficult, and may require a significant amount of time. Recall from Figure 4 that zero frequency components show up in the Gabor transform also, and that a non-vanishing zero-frequency component implies a change in system steady state as a result of the fault. With the aid of the signal model it is possible to predict that a zero frequency component will disappear before all dynamic behavior in the signal is damped out. Because hypothesis refinement is based on the elimination of hypotheses that become inconsistent with the measurement data, the absence of a zero-frequency component can be used to eliminate hypotheses that fault hypotheses linked to dissipative elements.

4. Discussion and Conclusions

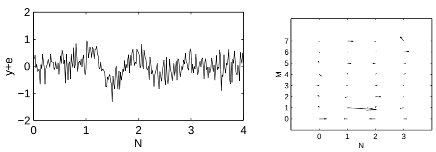
This paper has outlined a more sophisticated transient detection and analysis methodology for TRANSCEND. In addition, detection of the direction of change and discontinuous changes aids in the initial fault isolation. The Gabor transform provides a direct mapping to some features of transients resulting from discrete changes in component parameter values in linear dynamic systems. Using a detector based on statistical signal processing techniques makes the qualitative fault isolation method of TRANSCEND ro-



(a) $y = e^{-at}sin(\omega t)$.



(b) $y = -e^{-at}sin(\omega t)$



(c) $y = e^{-at}cos(\omega t)$

Figure 5. Transient signals that differ only in sign or phase. Signals (left) are shown with additive Gaussian noise (σ=0.3), $f = 1, t_{Arr} = 1$, Transform coefficients (right) are shown as complex valued data in a quiver plot.

bust to noise, and improves the sensitivity to faults that result from small parameter changes.

Improvements on the current method are possible both with respect to the fault detection problem as well as the fault isolation problem. Analysis of the Gabor transform has shown that the representation, is quite sensitive to non-integer arrival times or frequencies, that is, misalignment of the signal with the TF grid. The oversampled Gabor transform may be used to gain higher resolution in either the time domain or frequency domain or both [5]. However, excessive oversampling would lead to a substantially larger computational effort.

Finding the optimal signal representation for fault isolation is an ongoing topic of research. A comparison of various linear TF transform domain detectors by Friedlander and Porat [4] showed that the Gabor based detector performs better than a Wavelet transform based detector for narrow band transient signals. However, further study is needed to determine how additional discriminating features derived from a different transform representation would improve the fault isolation task, possibly at some loss of sensitivity. The ability to derive some qualitative transient features depends on the signal representation used. For example, the detection of discontinuous changes in the wavelet domain is typically realized by interpreting those changes as local high frequency components (singularity detection). Finally, it should be noted that a linear TF transform based detector belongs to the class of matched subspace detectors. A realization of the Gabor transform detector as a matched subspace detector in the data domain was given in [13].

Acknowledgments

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