Discrete-Time IDA-Passivity Based Control of Coupled Tank Processes Subject To Actuator Saturation

Nicholas Kottenstette, Joseph Porter, Gabor Karsai, Janos Sztipanovits *
*ISIS/Vanderbilt University

NOTICE: this is the author's version of a work that was accepted for publication in the 3rd International Symposium on Resilient Control Systems August 10-12, 2010 in Idaho Falls, Idaho (ISRCS 2010). Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication.
***Abstract*—Interconnection damping assignment passivity based control (IDA-PBC) is an emerging control design method which allows an engineer to systematically design an advanced controller for complex non-linear systems. As a result specific gain ranges can be determined which can prevent an operator (adversary) from accidentally (maliciously) setting control gains which could potentially destabilize the system. However in order to generate the controller the engineer will have to resort to using symbolic numerical solvers in order to complete the design. This can be both a cumbersome and error-prone task which can be automated. We present initial results of a tool which simplifies IDA-PBC. In addition many fluid control problems posses tight operating regions in which pumps degrade over time. As a result actuator saturation may occur for given set-point profiles which will lead to integrator wind-up and more oscillatory behavior. We provide a non-linear anti-windup control-law which greatly improves system resilience to such degradation. Finally we demonstrate that IDA-PBC works reasonably well for moderately large sampling times by simply applying the bilinear transform to approximate any additional (non-linear) integral control terms.**

I. Introduction

The design of resilient control systems necessitates novel developments at the intersection of computer science and control theory. The control of complex dynamic systems is a well-studied area, but much less is known about how to implement such control systems that are able to tolerate shortcomings of non-ideal software and network-based implementation platforms. Additionally, not only implementation side-effects have to be mitigated, but also potential issues related to security of the control system. For instance, if an operator's interface is compromised, will the attacker be able to set control gains in such a manner so as to destabilize the system? If an actuator is degrading or has been compromised, will the operator be able to quickly identify and quantify such a system fault? Many processes are highly non-linear and quite difficult to control, as a result typical linear approximations are often made and the resulting "safe" operating range is quite narrow. Interconnection damping assignment passivity based control (IDA-PBC) is an emerging method to systematically tackle the design of highly non-linear systems and derive intuitive control laws typically with many linear control-law elements and reasonable tuning gain ranges to allow an operator more flexibility in tuning a given system and to identify system degradation, all while being able to safely limit the operator from accidentally introducing destabilising gains.

Passivity is a mathematical property of the controller implementation, and could be realized in different ways. The approach described here applies to a large family of physical systems which can be described by both linear and non-linear system models [1]–[3], including systems which can be described by cascades of passive systems such as quad-rotor aircraft [4]. Most recently IDA-PBC has been shown to be effective in determining controllers which render non-minimum phase systems (which are typically very difficult to control) to be dissipative and asymptotically stable [5], [6].

A classic, but challenging control problem related to process control is the four tank process which can exhibit both minimum and nonminimum-phase behavior by simply changing valve flow ratios [7], [8]. In [8] it is first shown how to derive a proportional-IDA-PBC-law to control a less complex two tank process before deriving a control law for the four tank process. We continue the study of the two tank process by introducing an integrator term to account for system uncertainty while introducing an integrator anti-windup compensator similar to that used to control thermal systems [9].

Section II provides an overview on IDA-PBC and an effective way to implement an anti-windup controller in order to improve system resilience. Section III recalls the system dynamics used to model the flow for coupled tank systems in addition it includes the Hamiltonian chosen to generate our control laws. Section V provides simulated results demonstrating resilience to actuator degradation and improvement in reducing oscillations with anti-windup control. Section VI provides conclusions for this paper.

II. IDA-PBC, Integrator Anti-Windup & Discrete Time Control

Our primary concern is to determine a desired state trajectory x^* which may be a function of a smaller subset of desired *independently-controllable* states x_{c}^* and a corresponding control law $u = \beta(x, x_{\mathsf{I}}, x^*) \in \mathbb{R}^m$ ($x, x^* \in \mathbb{R}^n$ and $x_{\mathsf{c}}^*, x_{\mathsf{I}} \in \mathbb{R}^p$ in which $p \leq n$ is the number of independently controllable states) for the following input-affine system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

augmented with p additional (non-linear) integrator states x_{I} to account for system uncertainty such that

$$\begin{aligned}\dot{x}_{\mathsf{I}} &= k_{\mathsf{I}}\left(f_{\mathsf{I}}(x_{\mathsf{c}}) - f_{\mathsf{I}}(x_{\mathsf{c}}^*)\right) \\ &= \mathrm{diag}\{k_{\mathsf{I}1}\left(f_{\mathsf{I}1}(x_{\mathsf{c}1}) - f_{\mathsf{I}1}(x_{\mathsf{c}1}^*)\right), \ldots, k_{\mathsf{I}p}\left(f_{\mathsf{I}p}(x_{\mathsf{c}p}) - f_{\mathsf{I}p}(x_{\mathsf{c}p}^*)\right)\}.\end{aligned}$$

N.B. it may not be necessary to introduce p integrators in order to account for system uncertainty, however, in order to

0Contract/grant sponsor (number): NSF (NSF-CCF-0820088)
Contract/grant sponsor (number): Air Force (FA9550-06-1-0312).

simplify discussion we will assume that the system studied is asymptotically stable for each controllable state. This will typically require an additional integral term in order to account for system uncertainty and actuator degradation. In addition, we desire to implement an anti-windup control law which will improve system resilience when either the control actuator deteriorates or other system parameters deviate substantially such that actuator saturation occurs. Finally, we consider a discrete-time implementation of our control-law in which we use a bilinear-transform to approximate the integral and anti-windup control law. Section II-A provides an overview of IDA-PBC which will allow us to effectively control (non)-linear systems and achieve large operating ranges while being able to quantify system degradation. Section II-B presents our integrator anti-windup compensator which typically improves system resilience. Finally, Section II-C presents our discrete-time implementation which we found to work exceptionally well by allowing large sampling times T_s.

A. IDA-PBC

IDA-PBC is concerned with rendering our input affine system (1) with augmented integrator control law to have the following final form in terms of the gradient of the desired Hamiltonian $H_d(x, x_\mathsf{I}, x^*)$:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_\mathsf{I} \end{bmatrix} = Q_d(x, x_\mathsf{I}) \nabla H_d(x, x_\mathsf{I}, x^*) \tag{2}$$

in which $Q_d(x, x_\mathsf{I}) \in \mathbb{R}^{(n+p)\times(n+p)}$ is negative definite ($Q_d(x, x_\mathsf{I}) < 0$) and the operation $\nabla = [\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}, \frac{\partial}{\partial x_{\mathsf{I}1}}, \dots, \frac{\partial}{\partial x_{\mathsf{I}p}}]^\mathsf{T}$. In order to guarantee that $x = x^*$ and $x_\mathsf{I} = 0$ at steady-state (for the ideal model-matching case) the control law $\beta(x, x_\mathsf{I}, x^*)$ should guarantee that:

$$\nabla H_d(x = x^*, x_\mathsf{I} = 0, x^*) = 0 \text{ necessary}$$
$$\nabla^2 H_d(x = x^*, x_\mathsf{I} = 0, x^*) > 0 \text{ a sufficient condition.}$$

Although not required for controller synthesis, many physical systems, such as robotic systems $Q_d = J_d - R_d$ in which $J_d = -J_d^\mathsf{T}$ is a skew-symmetric matrix representing the underlying network structure of the system whereas $R_d = R_d^\mathsf{T} \geq 0$ describes the damping in the system [10]. Such observations may prove useful in choosing an initial Hamiltonian to begin control design, however, by choosing Q_d to simply be a constant negative definite matrix an engineer can systematically determine a controller $\beta(x, x_\mathsf{I}, x^*)$ as originally described in [8] and summarized here (in which we add some additional discussion on determining x^* from x_c^* while improving upon the discussion in introducing additional integrator terms).

1) Recall that the introduction of p additional (non-linear) integrators results in an augmented state-space description in order to account for system uncertainty:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_\mathsf{I} \end{bmatrix} = \begin{bmatrix} f(x) \\ k_\mathsf{I}\left(f_\mathsf{I}(x_\mathsf{c}) - f_\mathsf{I}(x_\mathsf{c}^*)\right) \end{bmatrix} + \begin{bmatrix} g(x) \\ 0 \end{bmatrix} u.$$

2) Select a candidate Hamiltonian $H(x, x_\mathsf{I})$ which depends on additional scaling terms k_i, $i \in \{1, \dots, n\}$ such that

$$\begin{bmatrix} \dot{x} \\ \dot{x}_\mathsf{I} \end{bmatrix} = Q_d \nabla H(x, x_\mathsf{I}) = \begin{bmatrix} f(x) \\ k_\mathsf{I}\left(f_\mathsf{I}(x_\mathsf{c}) - f_\mathsf{I}(x_\mathsf{c}^*)\right) \end{bmatrix}$$

in which $Q_d \in \mathbb{R}^{(n+p)\times(n+p)}$ is a constant matrix.

3) Determine the conditions on k_i and k_I such that Q_d is negative definite. We do this by verifying that the negative of the Hermitian of Q_d ($-\operatorname{He}\{Q_d\} = -\frac{1}{2}(Q_d^\mathsf{T} + Q_d)$) is positive definite using Sylvester's Criterion (the determinants of the leading principal submatrices of $-\operatorname{He}\{Q_d\}$ are positive).

4) Determine a matrix $P \in \mathbb{R}^{(n+p)\times m}$ having columns spanning the null-space of $g^\perp Q_d$ ($g^\perp Q_d P = 0$ and normalized such that $g^\dagger Q_d P = -I$) in which $g^\perp(x) \in \mathbb{R}^{(n-m)\times(n+p)}$ is the maximum rank left annihilator of $g(x)$ such that $g^\perp(x)[g(x)^\mathsf{T}, 0^\mathsf{T}]^\mathsf{T} = 0$ and $g^\dagger(x) \in \mathbb{R}^{m\times(n+p)}$ is the left-inverse of $g(x)$ such that $g^\dagger(x)[g(x)^\mathsf{T}, 0^\mathsf{T}]^\mathsf{T} = I$ (P is used to compute the characteristic $z = P^\mathsf{T}[x^\mathsf{T}, x_\mathsf{I}^\mathsf{T}]^\mathsf{T}$).

5) Denoting $\tilde{x} = [(x - x^*)^\mathsf{T}, (x_\mathsf{I} - 0)^\mathsf{T}]^\mathsf{T}$, $\tilde{z} = P^\mathsf{T}\tilde{x}$ and choosing the desired Hamiltonian H_d to be of the following form:

$$H_d(x, x_\mathsf{I}, x^*) = H(x, x_\mathsf{I}) + \frac{1}{2}\tilde{z}^\mathsf{T} Q \tilde{z} + l^\mathsf{T} z$$

in which $Q \in \mathbb{R}^{m\times m}$ $Q = Q^\mathsf{T} > 0$. The corresponding gradient is

$$\nabla H_d(x, x_\mathsf{I}, x^*) = \nabla H(x, x_\mathsf{I}) + PQP^\mathsf{T}\tilde{x} + Pl.$$

When $x = x^*$ and $x_\mathsf{I} = 0$ the gradient simplifies to

$$\nabla H_d(x = x^*, x_\mathsf{I} = 0, x^*) = \nabla H(x = x^*, x_\mathsf{I} = 0) + Pl$$

which we use to determine l and the uncontrolable parts of x^* which are not part of x_c^* such that $\nabla H_d(x = x^*, x_\mathsf{I} = 0, x^*) = 0$. In addition we verify that the corresponding Hessian is positive definite for $x = x^*$ and $x_\mathsf{I} = 0$

$$\nabla^2 H_d(x = x^*, x_\mathsf{I} = 0, x^*) = \nabla^2 H(x = x^*, x_\mathsf{I} = 0) + PQP^\mathsf{T}.$$

6) Finally we determine our control law $\beta(x, x_\mathsf{I}, x^*)$:

$$\beta(x) = g^\dagger \left\{ Q_d \nabla H_d(x, x_\mathsf{I}, x^*) - \begin{bmatrix} f(x) \\ k_\mathsf{I}\left(f_\mathsf{I}(x_\mathsf{c}) - f_\mathsf{I}(x_\mathsf{c}^*)\right) \end{bmatrix} \right\}$$
$$\beta(x, x_\mathsf{I}, x^*) = g^\dagger \left\{ Q_d P \left(QP^\mathsf{T}\tilde{x} + l\right) \right\}$$

Since P was normalized such that $g^\dagger Q_d P = -I$ then our final control law has the following simplified form:

$$\beta(x, x_\mathsf{I}, x^*) = -K_p \tilde{x} + u^*$$
$$\text{in which } K_p = QP^\mathsf{T},\ u^* = -l.$$

B. Integrator Anti-Windup Compensator

In [8] the authors addressed integrator wind-up issues by simply setting $\dot{x}_{\mathsf{I}} = 0$ if $\beta(x, x_{\mathsf{I}}, x^*) > u_{\max}$ or $\beta(x, x_{\mathsf{I}}, x^*) < u_{\min}$. Unfortunately this ad hoc solution caused the simulation to halt when evaluating their control law for the two tank and four tank processes when actuator saturation occurred. When the overall system to be controlled is asymptotically stable a more effective approach is to introduce an additional feedback term which attempts to approximate the system dynamics as if saturation has not occurred [9]. Since the integrator dynamics can not describe the case when $x < 0$ we will use a linear approximation of the system with respect to the desired trajectory components x^*. In addition the matrix $g(x)$ for the systems studied do not depend on x. We denote the Jacobian of $f(x)$ as $A(x)$ such that

$$A(x) = \begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \cdots & \frac{\partial f_1(x)}{\partial x_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n(x)}{\partial x_1} & \cdots & \frac{\partial f_n(x)}{\partial x_n} \end{bmatrix}.$$

Specifically we modify the integrator dynamics as follows:

$$\dot{x}_{\mathsf{I}} = k_{\mathsf{I}} \left(f_{\mathsf{I}}(x_{\text{linc}}) - f_{\mathsf{I}}(x_c^*) \right), \; x_{\text{linc}} = \bar{x}_c + x_c$$

in which x_{linc} is further constrained to gaurantee that $f_{\mathsf{I}}(x_{\text{linc}})$ is valid (ie. if $f_{\mathsf{I}}(x_{\text{linc}}) = \sqrt{x_{\text{linc}}}$ then if $x_{\text{linc}} < 0$ set $x_{\text{linc}} = 0$). $\bar{x}_c$ are the appropriate components of $\bar{x}$ which are determined from the following dynamic relationship

$$\dot{\bar{x}} = A(x^*)\bar{x} + g\bar{u}$$
$$\bar{u} = u - \text{sat}(u, u_{\min}, u_{\max}), \; u = \beta(x, x_{\mathsf{I}}, x^*)$$
$$\text{sat}(u, u_{\min}, u_{\max}) = \begin{cases} u \text{ if } u_{\min} \le u \le u_{\max}, \\ u_{\min} \text{ if } u < u_{\min}, \\ u_{\max} \text{ otherwise.} \end{cases}$$

We observe that for the linear time-invariant (*LTI*) case that if exact knowledge of the plant dynamics are given such that both $A(x)x = f(x)$ and the exact saturation model is known then the closed-loop dynamics considered for stability are identical to those considered when actuator saturation is not considered. Therefore stability is unaffected using our proposed control scheme for the *LTI* case. Analysis for the non-linear case is clearly much more involved and worthy of future study. However, for the non-linear tank processes studied, using the jacobian $A(x)$ to approximate the plant dyanmics was sufficient to prevent integrator wind-up while maintaining stability.

C. Discrete-Time Implementation

Observing the final control-law for $\beta(x, x_{\mathsf{I}}, x^*)$ and noting the form for the actuator anti-windup structure in computing $\dot{\bar{x}}$ and $\dot{x}_{\mathsf{I}}$. Most of the control components consists of a standard matrix multiplication and the corresponding (non-linear) integral terms can be approximated by applying either a matched pole-zero or bilinear transform [11]. The bilinear transform is preferred because it preserves the passivity properties of the integration term which typically allows for longer sample and hold times than if a matched pole-zero method was used. Thus reducing communication bandwidth and typically reducing sensitivity to time delay jitter. We note that the integer k shall be related to time t and sampling rate $T_s > 0$ as follows $k = \lfloor \frac{t}{T_s} \rfloor$ for all $t \ge 0$. Specifically, our discrete-time implementation is as follows:

$$u(k) = \beta(x(kT_s), x_{\mathsf{I}}(k-1), x^*(k))$$
$$\bar{u}(k) = u(k) - \text{sat}(u(k), u_{\min}, u_{\max})$$
$$\dot{\bar{x}}(k) = A(x^*(k))\bar{x}(k) + g\bar{u}(k)$$
$$\bar{x}(k) = \bar{x}(k-1) + \frac{T_s}{2}\left[\dot{\bar{x}}(k) + \dot{\bar{x}}(k-1)\right]$$
$$x_{\text{linc}}(k) = \bar{x}_c(k) + x_c(kT_s) \in [x_{\min}, x_{\max}]$$
$$\dot{x}_{\mathsf{I}}(k) = k_{\mathsf{I}}\left[f_{\mathsf{I}}(x_{\text{linc}}(k)) - f_{\mathsf{I}}(x_c^*(k))\right]$$
$$x_{\mathsf{I}}(k) = x_{\mathsf{I}}(k-1) + \frac{T_s}{2}\left[\dot{x}_{\mathsf{I}}(k) + \dot{x}_{\mathsf{I}}(k-1)\right]$$

in which $u(t) = u(k), \; t \in [kT_s, (k+1)T_s)$. It is of future interest to implement an observer for the plant-subsystem in order to preserve dissipative properties in the discrete-time setting [12]. Such an observer set-up could potentially lead to a one-to-one mapping between the continuous-time and discrete-time implementation in regards to satisfying discrete-time stability for a given set of gain constraints.

III. IDA-PBC of Coupled Tank Systems

A classic problem in the process control laboratory is to learn how to control the height of columns of water for either a coupled two tank [13] or four tank process [7]. Each problem is particularly interesting in that there are only half as many actuators (pumps) as there are tanks of water whose heights there are to control. Therefore each process is under-actuated in which only half of the tanks heights can be independently controlled and in particular for the four tank process it is unclear how to systematically apply methods such as back-stepping control to the process [14]. However, as [8] has demonstrated IDA-PBC shows to be a promising tool to handle such a complicated system. In applying IDA-PBC to the four tank process we discovered some *additional restrictions* are required on the independent controllable states x_c and the control gains k_i and k_{I}. Since we are particularly interested in implementing a resilient integrator anti-windup compensator Section III-A recalls the coupled two tank process model which [8] modified to split the flow into both the upper and lower tanks. We will extend the discussion on the control of this system by considering an additional integral term to account for model uncertainty. Section III-B will recall the coupled four tank process model.

A. Two Tank Process

The two tank process consists of a single gear-pump which provides volumetric flow of a fluid from a bottom-reservoir proportional to the control input u whose flow is then split such that ideally γu is sent to a lower-tank with cross-sectional area A_1 and drain-orifice area a_1 which drains back into the bottom-reservoir. The remaining $(1 - \gamma)u$ amount of fluid is sent to the upper-tank with cross-sectional area A_2 and drain-orifice area a_2 which drains back into the lower-tank. The heights of the fluid in lower and upper-tanks is denoted x_1 and x_2 respectively. Using Torricelli's Law the system dynamics have the following form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{-a_1\sqrt{2gx_1}+a_2\sqrt{2gx_2}}{A_1}\\ -\frac{a_2\sqrt{2gx_2}}{A_2}\end{bmatrix} + \begin{bmatrix}\frac{\delta_\gamma \gamma}{A_1}\\ \frac{1-\delta_\gamma\gamma}{A_2}\end{bmatrix} k_u u. \tag{3}$$

For simplicity of discussion k_u and δ_γ are nominally considered to be equal to one, however we will perturb these values in order to consider effects including pump degradation ($k_u < 1$) and uncertainty in the flow-ratio $\gamma \in (0,1)$ such that $0 < k_\gamma < \frac{1}{\gamma}$. It should be clear from (3) that only the height of one of the tanks can be independently controlled. We will control the height of the lower-tank $x_1 = x_c$ and determine x_2^* based on our desired height x_1^* and the system dynamics. In order to account for system uncertainty we include the following integral control term:

$$\dot{x}_{\text{I}1} = a_1\sqrt{2g}\left(\sqrt{x_1^*} - \sqrt{x_1}\right). \tag{4}$$

The corresponding Hamiltonian used to generate our controller is

$$H(x) = \sum_{i=1}^{2} \frac{2}{3} k_i a_i \sqrt{2g} x_i^{\frac{3}{2}} + k_{\text{I}1} a_1 \sqrt{x_1^*} x_{\text{I}1}. \tag{5}$$

Which results in:

$$Q_d = \begin{bmatrix} -\frac{1}{A_1 k_1} & \frac{1}{A_1 k_2} & 0\\ 0 & -\frac{1}{A_2 k_2} & 0\\ \frac{k_{\text{I}1}}{k_1} & 0 & -\frac{k_{\text{I}1}}{k_1}\end{bmatrix}$$

which is positive-definite iff $0 < k_1 < \frac{(4-A_1 k_{\text{I}1})A_1 k_2}{A_2}$, $0 < k_2 < \infty$ and $0 < k_{\text{I}1} < \frac{4}{A_1}$; and the remaining control related terms $P = [k_1, k_2(1-\gamma), k_1]^{\mathsf{T}}$, $l = -a_1\sqrt{2gx_1}$ and $x_2^* = \frac{a_1^2(1-\gamma)^2}{a_2^2} x_1^*$.

B. Four Tank Process

The four tank process consists of: i) lower-tanks Tank 1 and Tank 2 with respective fluid height x_1 and x_2 which we wish to control $x_c = [x_1, x_2]^{\mathsf{T}}$; ii) upper-tanks Tank 3 and Tank 4 with fluid height x_3 and x_4 such that $x = [x_c^{\mathsf{T}}, x_3, x_4]^{\mathsf{T}}$; iii) two gear pumps Pump 1 and Pump 2 generating volumetric flows u_1 and u_2 respectively such that $u = [u_1, u_2]^{\mathsf{T}}$; iv) Valve 1 which splits the flow to Tank 1 ($\gamma_1 u_1$) and Tank 4 ($(1-\gamma_1)u_1$); v) Valve 2 which splits the flow to Tank 2 ($\gamma_2 u_2$) and Tank 3 ($(1-\gamma_2)u_2$); and vi) Tank 3 drains into Tank 1 whereas Tank 4 drains into Tank 2. This cross-coupling creates a system which can be either minimum phase $1 < (\gamma_1 + \gamma_2) < 2$ or nonminimum-phase $0 < (\gamma_1 + \gamma_2) < 1$. Using Torricelli's Law the system dynamics for the four tank process are as follows

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}\frac{-a_1\sqrt{2gx_1}+a_3\sqrt{2gx_3}}{A_1}\\ \frac{-a_2\sqrt{2gx_2}+a_4\sqrt{2gx_4}}{A_2}\\ -\frac{a_3\sqrt{2gx_3}}{A_3}\\ -\frac{a_4\sqrt{2gx_4}}{A_4}\end{bmatrix} + \begin{bmatrix}\frac{\delta_{\gamma 1}\gamma_1}{A_1} & 0\\ 0 & \frac{\delta_{\gamma 2}\gamma_2}{A_2}\\ 0 & \frac{1-\delta_{\gamma 2}\gamma_2}{A_3}\\ \frac{1-\delta_{\gamma 1}\gamma_1}{A_4} & 0\end{bmatrix} k_u \begin{bmatrix}u_1\\ u_2\end{bmatrix}. \tag{6}$$

As was done in [8] we choose the following integral control states

$$\begin{bmatrix}\dot{x}_{\text{I}1}\\ \dot{x}_{\text{I}2}\end{bmatrix} = \begin{bmatrix}k_{\text{I}1} a_1\sqrt{2g}\left(\sqrt{x_1^*} - \sqrt{x_1}\right)\\ k_{\text{I}2} a_2\sqrt{2g}\left(\sqrt{x_2^*} - \sqrt{x_2}\right)\end{bmatrix}. \tag{7}$$

and corresponding Hamiltonian

$$H(x) = \sum_{i=1}^{4} \frac{2}{3} k_i a_i \sqrt{2g} x_i^{\frac{3}{2}} + \sum_{j=1}^{2} k_{\text{I}j} a_j \sqrt{x_j^*} x_{\text{I}j}. \tag{8}$$

Which results in:

$$Q_d = \begin{bmatrix} \frac{-1}{A_1 k_1} & 0 & \frac{1}{A_3 k_3} & 0 & 0 & 0\\ 0 & \frac{-1}{A_2 k_2} & 0 & \frac{1}{A_2 k_4} & 0 & 0\\ 0 & 0 & \frac{-1}{A_3 k_3} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{-1}{A_3 k_3} & 0 & 0\\ \frac{k_{\text{I}1}}{k_1} & 0 & 0 & 0 & -\frac{k_{\text{I}1}}{k_1} & 0\\ 0 & \frac{k_{\text{I}2}}{k_2} & 0 & 0 & 0 & -\frac{k_{\text{I}2}}{k_2}\end{bmatrix}$$

which is positive-definite iff

$$0 < k_1 < \frac{(4 - k_{\text{I}1}A_1)\,k_3}{A_3},\ 0 < k_2 < \frac{(4 - k_{\text{I}2}A_2)\,k_4}{A_4}$$
$$0 < k_3,\ k_4 < \infty,\ 0 < k_{\text{I}1} < \frac{4}{A_1},\ 0 < k_{\text{I}2} < \frac{4}{A_2};$$

and the remaining control related terms

$$P = \begin{bmatrix}\gamma_1 k_1 & (\gamma_2 - 1)k_1\\ (1-\gamma_1)k_2 & \gamma_2 k_2\\ 0 & (1-\gamma_2)k_3\\ (1-\gamma_1)k_4 & 0\\ \gamma_1 k_1 & (1-\gamma_2)k_1\\ (1-\gamma_1)k_2 & \gamma_2 k_2\end{bmatrix},$$

$$\begin{bmatrix}l_1\\ l_2\end{bmatrix} = \begin{bmatrix}-\frac{\gamma_2 a_1\sqrt{2gx_1^*}+(\gamma_2-1)a_2\sqrt{2gx_2^*}}{\gamma_1+\gamma_2-1}\\ -\frac{(\gamma_1-1)a_1\sqrt{2gx_1^*}+\gamma_1 a_2\sqrt{2gx_2^*}}{\gamma_1+\gamma_2-1}\end{bmatrix},\ \begin{bmatrix}x_3^*\\ x_4^*\end{bmatrix} = \begin{bmatrix}\frac{(1-\gamma_2)^2}{2ga_3^2} l_2^2\\ \frac{(1-\gamma_1)^2}{2ga_4^2} l_1^2\end{bmatrix}.$$

N.B. it appears that in [8] the authors incorrectly applied Sylvester's Criterion directly to $-Q_d$ instead of the negative of the Hermitian of Q_d ($-\mathrm{He}\{Q_d\}$) which resulted in their clearly incorrect constraints for the controller gains k_i and k_{I} for the four tank process. Finally our analysis revealed that x_1^* and x_2^* can not be set completely independent of each other. Specifically in order for $\nabla H_d(x = x^*, x_{\text{I}} = 0, x^*) = 0$ then

$$\left(\frac{a_1\gamma_2}{a_2(1-\gamma_2)}\right)^2 < \frac{x_2^*}{x_1^*} < \left(\frac{a_1(1-\gamma_1)}{a_2\gamma_1}\right)^2 \text{ if } (\gamma_1+\gamma_2) < 1$$
$$\left(\frac{a_1(1-\gamma_1)}{a_2\gamma_1}\right)^2 < \frac{x_2^*}{x_1^*} < \left(\frac{a_1\gamma_2}{a_2(1-\gamma_2)}\right)^2 \text{ if } (\gamma_1+\gamma_2) > 1.$$

IV. Symbolic Analysis

We created a prototype tool which symbolically derives the controller expressions using the dynamic model and some user guidance. We used the MuPAD symbolic analysis tool available in Matlab [15]. Our tool follows the steps described in section II in which we highlight some additional details in regards to our implementation.

1) For each leading principal submatrix $-\mathrm{He}\{Q_d\}_k \in \mathbb{R}^{k\times k}$ $k \in \{1,\ldots,n+p\}$ we formed the symbolic

determinant expression $|-\mathrm{He}\{Q_d\}_k|$. Then use MuPAD's `solve()` command to jointly solve for the control coefficients k_i, k_I subject to the constraint that $|-\mathrm{He}\{Q_d\}_k| > 0$.

2) The calculation of P is a bit more involved:
   a) Use MuPAD's function `linalg::nullspace()` to extract the nullspace basis set for $g(x)^T$, then concatenate the resulting list of vectors to construct the matrix $g^\perp(x)$.
   b) Compute $g^\dagger$ using the Moore-Penrose left psuedoinverse $g^\dagger = (g(x)^T g(x))^{-1} g(x)^T$.
   c) Form the matrices $g(x)^\perp Q_d$ and $g(x)^\dagger Q_d$. The matrix P_null comes from the basis vectors for the null space of $g(x)^\perp Q_d$. Solving $P = -P_\text{null}(g^\dagger Q_d P_\text{null})^{-1}$ yields the appropriately scaled matrix P.

V. Simulation Results

In evaluating our proposed solution we shall take a closer look at system performance for the modified two tank process described in [8]. Specifically we will compare our controller with the additional integrator and corresponding integrator anti-windup compensator to the original proportional feedback controller presented in [8]. We will see that the integrator is able to effectively compensate for both actuator degradation and flow-ratio uncertainty. In addition the integrator anti-windup compensator works sufficiently well in prohibiting significant oscillatory behavior when operating at the systems limits. Next we will compare IDA-PBC performance to the decentralized controllers presented in [7] for both the minimum and non-minimum phase cases. We will see that the IDA-PBC is both comparable for the minimum phase case while being vastly superior for the non-minimum phase case.

A. Two Tank Process

For the two tank process the system operating parameters are as follows: $A_1 = 50.3$ cm^2, $A_2 = 28.3$ cm^2, $a_1 = .233$ cm^2, $a_2 = .127$ cm^2, $\gamma = .4$, $\delta_\gamma = .75$, $k_u = .75$, $u_\text{min} = 0$, $u_\text{max} = 100$, $x_1(0) = 15$ cm, $x_2(0) = \left(\frac{(1-\delta_\gamma\gamma)a_1}{a_2}\right)^2 x_1(0)$, $T_s = 1$ second and $g = 981$ cm/s^2. Set-point trajectories for both the two tank and four tank processes are smoothed using a discrete-time filter which results from applying the bilinear transform to the corresponding continuous time-time filter model $H_\text{traj}(s) = \frac{\omega_\text{traj}^2}{s^2+2\zeta_\text{traj}\omega_\text{traj}s+\omega_\text{traj}^2}$. We compared our controller for the two tank process to the controller presented in [8] which lacks the additional integrator term $x_{\text{I}1}$ to compensate for model uncertainty. Specifically $u = -[k_1, (1-\gamma)k_2]\hat{x} + a_1\sqrt{2gx_1^*}$ in which $k_1 = 10$ and $k_2 = 1.01\frac{A_2 k_1}{A_1 4} = 1.4206$.

B. Four Tank Process

The reference for comparison is the decentralized controller (DC) used to control the four tank process [7]. Specifically two PI-controllers were used such that $U_l(s) = K_l\left(1 + \frac{1}{T_{il}}s\right)(X_l^*(s) - X_l(s))$, $l \in \{1,2\}$ in which $(K_1 = 3.0, T_{i1} = 30)$ and $(K_2 = 2.7, T_{i2} = 40)$ for

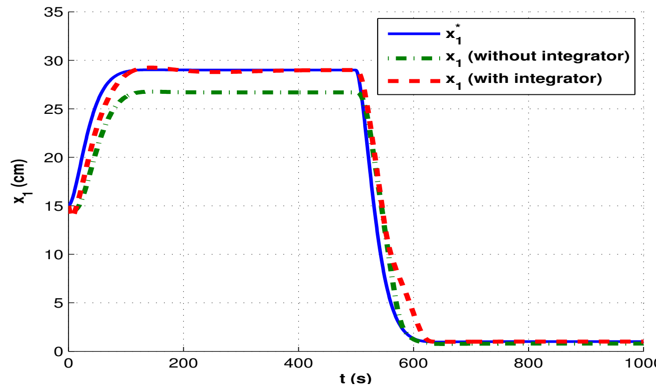

Fig. 1. Two tank process $x_1(t)$.

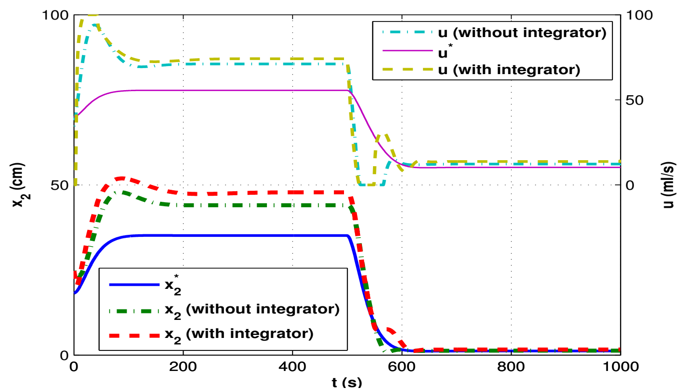

Fig. 2. Two tank process $x_2(t)$ and $u(t)$.

the minimum-phase case and $(K_1 = 1.5, T_{i1} = 110)$ and $(K_2 = -.12, T_{i2} = 220)$ for the nonminimum-phase case. The remaining parameters for the process are as follows: i) $A_1 = A_3 = 28$ cm^2, $A_2 = A_4 = 32$ cm^2, $a_1 = a_3 = 0.071$ cm^2, $a_2 = a_4 = 0.057$ cm^2, $u_{1\,\text{min}} = u_{2\,\text{min}} = 0$, $u_{1\,\text{max}} = u_{2\,\text{max}} = 100$, $k_u = 0.8$; ii) either $\gamma_1 = 0.7$, $\gamma_2 = 0.6$, $x_1(0) = 12.4$ cm, $x_2(0) = 12.7$ cm, $x_3(0) = 1.8$ cm, and $x_4(0) = 1.4$ cm for the minimum-phase case or $\gamma_1 = 0.43$, $\gamma_2 = 0.34$, $x_1(0) = 12.6$ cm, $x_2(0) = 13.0$ cm, $x_3(0) = 4.8$ cm, and $x_4(0) = 4.9$ cm for the nonminimum phase case.

For the minimum-phase example (Figs. 3 and 4) the IDA-PBC parameters are $k_3 = 100$, $k_4 = 100$, $\epsilon_1 = 0.75$, $\epsilon_2 = 0.75$, $\epsilon_{\text{I}1} = 0.4$, $\epsilon_{\text{I}2} = 0.4$, $T_s = 0.1$ s. For the nonminimum-phase example (Figs. 5 and 6) the IDA-PBC parameters are $k_3 = 5$, $k_4 = 5$, $\epsilon_1 = 0.75$, $\epsilon_2 = 0.75$, $\epsilon_{\text{I}1} = 0.75$, $\epsilon_{\text{I}2} = 0.75$. In which the remaining controller coefficients are computed using the following relationships $k_{\text{I}1} = \epsilon_{\text{I}1}\frac{4}{A_1}$, $k_{\text{I}2} = \epsilon_{\text{I}2}\frac{4}{A_2}$, $k_1 = \epsilon_1\frac{(4-k_{\text{I}1}A_1)k_3}{A_3}$ and $k_2 = \epsilon_2\frac{(4-k_{\text{I}2}A_2)k_4}{A_4}$.

VI. Conclusions

From Fig. 6 it is clear that IDA-PBC can achieve superior tracking performance when compared to the decentralized controller for the nonminimum-phase four tank process. Unlike the decentralized controller we evaluated, the IDA-PBC provides both explicit constraints on allowable controller gains

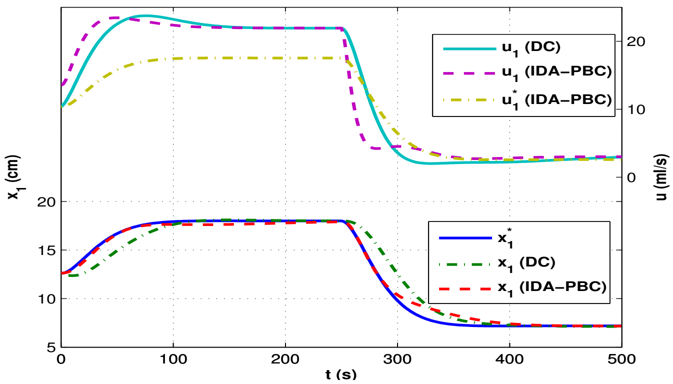

Fig. 3. Minimum-phase four tank process $x_1(t),\ u_1(t)$.

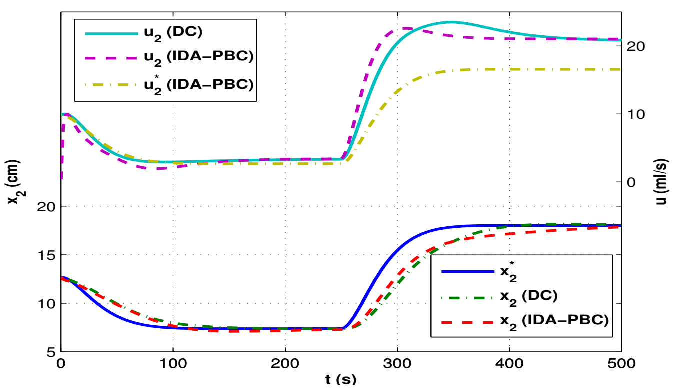

Fig. 4. Minimum-phase four tank process $x_2(t),\ u_2(t)$.

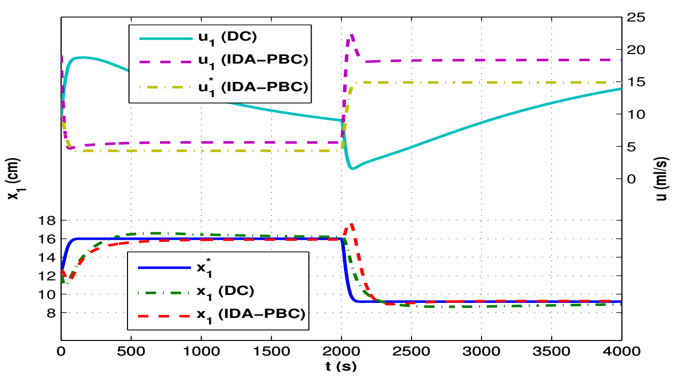

Fig. 5. Nonminimum-phase four tank process $x_1(t),\ u_1(t)$.

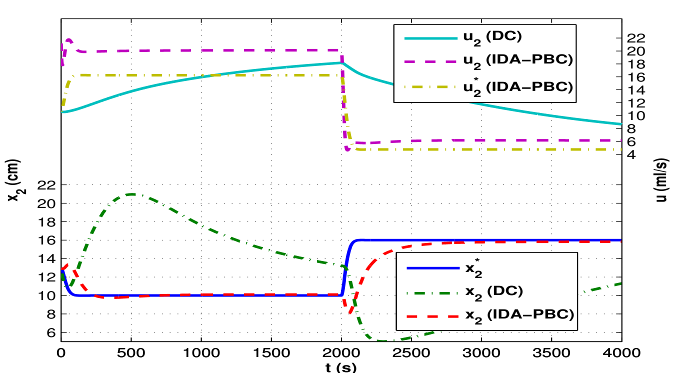

Fig. 6. Nonminimum-phase four tank process $x_2(t),\ u_2(t)$.

and set-point trajectories x^* which can be enforced at run-time. We clarified how to correctly determine if $Q_d < 0$ and verified correct constraints on k_i and k_I for the four tank process which were incorrectly determined in [8] which allowed us to provide new results showing a working controller for the non-minimum phase four-tank system. We further improved system resilience by implementing a feasible integrator anti-windup compensator as demonstrated in the full-scale step responses of the two-tank process depicted in Fig. 1 and Fig. 2. The explicit solution for $u^* = -l$ and the uncontrollable components of x^* clearly provide visual indications about actuator degradation and uncertainty in γ for the coupled tank processes we studied. Finally, we demonstrated that the bilinear transform can be used to achieve moderately large sample times $T_s = .1$ second which will reduce both computational and communication demands.

REFERENCES

[1] W. M. Haddad and V. S. Chellaboina, *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*. Princeton, New Jersey, USA: Princeton University Press, 2008.

[2] A. van der Schaft, *L2-Gain and Passivity in Nonlinear Control*. Secaucus, NJ, USA: Springer-Verlag New York, Inc., 1999.

[3] R. Ortega, A. Loria, P. J. Nicklasson, and H. Sira-Ramirez, *Passivity-Based Control of Euler-Lagrange Systems*. Great Britain: Springer-Verlag London Limited, 1998.

[4] N. Kottenstette and J. Porter, "Digital passive attitude and altitude control schemes for quadrotor aircraft," in *Control and Automation, 2009. ICCA 2009. IEEE Intl. Conf. on*, 12 2009, pp. 1761 –1768.

[5] F. Dörfler, J. Johnsen, and F. Allgöwer, "An introduction to interconnection and damping assignment passivity-based control in process engineering," *Journal of Process Control*, 2009.

[6] H. Ramírez, D. Sbarbaro, and R. Ortega, "On the control of non-linear processes: An IDA-PBC approach," *Journal of Process Control*, vol. 19, no. 3, pp. 405–414, 2009.

[7] K. H. Johansson, "The quadruple-tank process: a multivariable laboratory process with an adjustable zero," *IEEE Transactions on Control Systems Technology*, vol. 8, no. 3, pp. 456 –465, 5 2000.

[8] J. K. Johnsen and F. Allgöwer, "Interconnection and damping assignment passivity-based control of a four-tank system," in *Lagrangian and Hamiltonian Methods for Nonlinear Control 2006*. Springer, 2007.

[9] Y. Fu, N. Kottenstette, Y. Chen, C. Lu, X. D. Koutsoukos, and H. Wang, "Feedback thermal control for real-time systems," in *16th IEEE Real-Time and Embedded Technology and Application Symposium (RTAS 2010)*, 2010, p. 10. [Online]. Available: http://www2.ee.kth.se/conferences/cpsweek2010/PDF/RTAS/Feedback%20Thermal%20Control%20for%20Real-time%20Systems.pdf

[10] R. Ortega, A. Van Der Schaft, B. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.

[11] G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 5th ed. Prentice-Hall, 2006.

[12] S. Stramigioli, C. Secchi, A. J. van der Schaft, and C. Fantuzzi, "Sampled data systems passivity and discrete port-hamiltonian systems," *IEEE Transactions on Robotics*, vol. 21, no. 4, pp. 574 – 587, 2005. [Online]. Available: http://dx.doi.org/10.1109/TRO.2004.842330

[13] K. Astrom and A. Ostberg, "A teaching laboratory for process control," *IEEE Control Systems Magazine*, vol. 6, no. 5, pp. 37–42, 1986.

[14] M. Krstic, P. Kokotovic, and I. Kanellakopoulos, *Nonlinear and adaptive control design*. John Wiley & Sons, Inc. New York, NY, USA, 1995.

[15] I. T. MathWorks, "Matlab," *The Language of Technical Computing, Version 7.8*, 2009.