L_2^m-stable digital-control networks for multiple continuous passive plants. ⋆

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Abstract: A constructive method is presented in which L_2^m-stability can be guaranteed for networked control of multiple *passive* plants in spite of random time varying delays and data dropouts. The passive plants are interfaced to a wave variable based *passive sampler* (PS) and *passive hold* (PH) which allows a passive digital control network to be constructed. A *power junction* is used to facilitate the interconnection of multiple passive plants and passive digital controllers. The power junction preserves passivity by guaranteeing that the overall power input to the system is greater than or equal to the power leaving the system. There are numerous ways to implement the power junction including the *averaging power junction* and the *consensus power junction* which are studied in this paper. In particular, a detailed steady state analysis is provided which relates the corresponding controller inputs to the plants outputs. The construction of our digital control network is completed by interconnecting the digital controllers to an *inner-product equivalent sampler* and zero-order hold (*IPESH*) which allows us to prove L_2^m-stability.

Keywords: passivity, networked control, consensus, wave variables, digital control

1. INTRODUCTION

The primary goal of our research is to develop reliable wireless digital control networks Antsaklis and Baillieul (2007). In the past we have shown numerous results related to the control of a single continuous time *passive* plant with a single digital controller over a network. In particular we have shown how to create a l_2^m*-stable* digital control network for a continuous *passive* plant (Kottenstette and Antsaklis, 2007, Theorem 4) and built on this result to show that the controller can be run in an asynchronous manner (Kottenstette and Antsaklis, 2008b, Theorem 1). We have also shown how a continuous time stability result (L_2^m*-stability*) can be achieved with a *passive* digital controller by interfacing a continuous time *passive* plant to a passive sampler (PS) and passive hold (PH) (Kottenstette et al., 2008, Corollary 1). The key is to transmit control and sensor data in the form of *wave variables* over networks similar to those depicted in (Kottenstette and Antsaklis, 2007, Fig. 2). The use of *wave variables* allows the network controlled system to remain stable when subject to both fixed time delays and data dropouts. In addition, if duplicate wave variable transmissions are dropped, then the network will remain stable in spite of time varying delays.

Recently we have shown how this framework can be modified to control multiple discrete-time passive plants over a wireless network by using a *power junction* and also guarantee l_2^m-stability (Kottenstette et al., 2009, Theorem 1). We noted in (Kottenstette et al., 2009, Section II-B) that L_2^m-stability results can also be shown with a power junction if continuous time plants were interfaced to a PS and PH. The PS and PH framework, unlike other data-reduction techniques used in telepresence systems Hirche and Buss (2007), does not require the user to take digital waves and convert them back to a continuous-time signal. Figure 1 shows this proposed configuration. In this paper,

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based on this architecture we provide detailed analysis for different power junctions. In doing so, we introduce the *consensus power junction* which is based on a recent work related to passivity based synchronizing networks which use continuous time feedback Chopra and Spong (2006). In addition this paper presents the steady state responses of the plant outputs in regards to a set of steady state controller (and plant) inputs.

The *power junction* is an abstraction to interconnect wave variables from multiple controllers and plants such that the total power input is always greater than or equal to the total power output (Kottenstette et al., 2009, Definition 1). Interconnecting wave variables in a 'power preserving' manner has appeared in the telemanipulation literature to augment potential position drift by modifying one of the waves u_m in a *passive* manner (Niemeyer and Slotine, 2004, Fig. 9). Other abstractions to interconnect wave variables have also appeared in the wave digital filtering literature Fettweis (1986). A *consensus power junction* will be introduced in the paper which interconnects wave variables to plants in a manner similar to that discussed in Chopra and Spong (2006) except: i) a controller can be explicitly used to steer the plants to a desired set-point and ii) the information does not need to be transmitted as a continuous-time waveform.

This paper is a significant refinement of our earlier work including Kottenstette and Antsaklis (2008a) in which the *power junction* first appeared and was significantly refined in Kottenstette et al. (2009) for the discrete time case. In this paper we show how the (*consensus* or *averaging*) *power junction* in conjunction with a PS and PH makes it possible to allow m digital-controllers to control up to $n-m$ continuous-time-plants. We prove that such a network can be shown to be L_2^m*-stable* if all the interconnected plants and controllers are *strictly-output passive*. This paper, uses the PS and PH in conjunction with the power junction to construct a digital control system for multiple continuous time plants and controllers which can achieve L_2^m-stability. A complete steady-state analysis for the *averaging power junction* (Definition 2) is provided in this paper. We introduce the *consensus power junction* (Definition 3), show that it satisfies the conditions for the power junction (Lemma 4) and provide a steady state analysis (Theorem 16). The rest of the paper is organized as follows: i) Section 2 presents the necessary background and the main results that demonstrate the design of network control systems for multiple-continuous-plants interconnected to (multiple)-digital-controllers through the power junction (Section 2.2) and the PS and PH (Section 2.3) which are L_2^m stable (Section 2.4), a detailed steady-state response analysis (Section 2.5) . Detailed simulations and proofs are available in an extended version of this paper Kottenstette and Chopra (2009).

2. NETWORKED CONTROL DESIGN

2.1 Wave Variables

Networks of a *passive* plant and controller are typically interconnected using *power variables. Power variables* are generally denoted with an *effort* and *flow* pair (e_*,f_*) whose product is power. They are typically used to show the exchange of energy between two systems using *bond graphs* Breedveld (2006); Golo et al. (2003). However, when these *power variables* are subject to communication delays the communication channel ceases to be *passive* which leads to network instabilities. Scattering Anderson and Spong (1988) or their reformulation known as the wave variables allow *effort* and *flow* variables to be transmitted over a network while remaining *passive* when subject to arbitrary fixed time delays and data dropouts Niemeyer and Slotine (2004).

$$u_{pk}(t) = \frac{1}{\sqrt{2b}}(bf_{pk}(t) + e_{dck}(t)),\ k \in \{m+1,\dots,n\} \quad (1)$$

$$v_{pk}(t) = \frac{1}{\sqrt{2b}}(bf_{pk}(t) - e_{dck}(t)) \quad (2)$$

$$v_{cj}(i) = \frac{1}{\sqrt{2b}}(bf_{dpj}(i) - e_{cj}(i)),\ j \in \{1,\dots,m\} \quad (3)$$

$$u_{cj}(i) = \frac{1}{\sqrt{2b}}(bf_{dpj}(i) + e_{dpj}(i)) \quad (4)$$

(1) can be thought of as each sensor output in a *wave variable* form for each plant $G_{pk},\ k \in \{m+1,\dots,n\}$ depicted in Fig. 1. Likewise, (3) can be thought of as each command output in a *wave variable* form for each controller $G_{cj},\ j \in \{1,\dots,m\}$ depicted in Fig. 1. The symbol $i \in \{0,1,\dots\}$ depicts discrete time for the controllers, and the symbol $t \in \mathbb{R}$ denotes continuous time and the two are related to the sample and hold time (T_s) such that $t = iT_s$. (1) and (2) respectively satisfy the following equality $\forall\ k \in \{m+1,\dots,n\}$:

$$\frac{1}{2}(u_{pk}^{\mathsf{T}}(t)u_{pk}(t) - v_{pk}^{\mathsf{T}}(t)v_{pk}(t)) = f_{pk}^{\mathsf{T}}(t)e_{dck}(t) \quad (5)$$

Similarly, (3) and (4) respectively satisfy the following equality $\forall\ j \in \{1,\dots,m\}$:

$$\frac{1}{2}(u_{cj}^{\mathsf{T}}(i)u_{cj}(i) - v_{cj}^{\mathsf{T}}(i)v_{cj}(i)) = f_{dpj}^{\mathsf{T}}(i)e_{cj}(i). \quad (6)$$

Denote $I \in \mathbb{R}^{m_s \times m_s}$ as the identity matrix. When implementing the wave variable transformation the continuous time plant "outputs" ($u_{pk}(t), e_{dck}(t)$) are related to the corresponding "inputs" ($v_{pk}(t), f_{pk}(t)$) as follows (Fig. 1):

$$\begin{bmatrix} u_{pk}(t) \\ e_{dck}(t) \end{bmatrix} = \begin{bmatrix} -I & \sqrt{2b}I \\ -\sqrt{2b}I & bI \end{bmatrix} \begin{bmatrix} v_{pk}(t) \\ f_{pk}(t) \end{bmatrix} \quad (7)$$

Next, the discrete time controller "outputs" ($v_{cj}(i), f_{dpj}(i)$) are related to the corresponding "inputs" ($u_{cj}(i), e_{cj}(i)$) as follows (Fig. 1):

$$\begin{bmatrix} v_{cj}(i) \\ f_{dpj}(i) \end{bmatrix} = \begin{bmatrix} I & -\sqrt{\frac{2}{b}}I \\ \sqrt{\frac{2}{b}}I & -\frac{1}{b}I \end{bmatrix} \begin{bmatrix} u_{cj}(i) \\ e_{cj}(i) \end{bmatrix} \quad (8)$$

The *power junction* indicated in Fig. 1 by the symbol PJ has waves entering and leaving the power junction as indicated by the arrows. Waves leaving the controllers v_{cj} and entering the power junction v_j in which $j \in \{1,\dots,m\}$ have the following relationship

$$v_j(i) = v_{cj}(i - p_j(i))$$

in which $p_j(i)$ denotes the time varying delay in transmitting the control wave from 'controller-j' to the power junction. Next, the input wave to the plant v_{pk} is a delayed

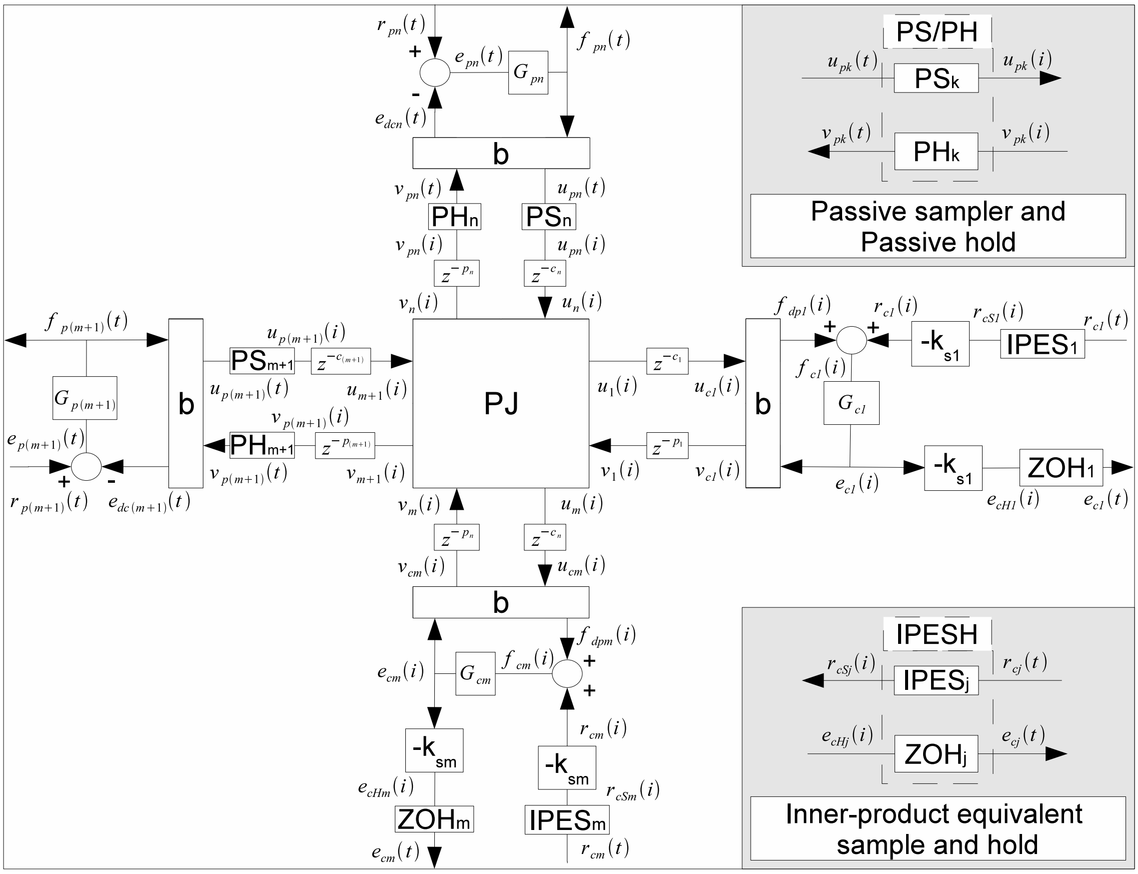

Fig. 1. A *power junction* control network.

version of the outgoing wave from the *power junction* $v_k,\ k \in \{m+1,\dots,n\}$ such that

$$v_{pk}(i) = v_k(i - p_k(i)),\ k \in \{m+1,\dots,n\}$$

in which $p_k(i)$ denotes the discrete time varying delay in transmitting the outgoing wave to 'plant-k'. In Fig. 1 the delays are represented as fixed for the discrete time case (i.e. z^{-p_k}). Next, the outgoing wave from each plant u_{pk} is related to the wave entering the power junction $u_k,\ k \in \{m+1,\dots,n\}$ as follows:

$$u_k(i) = u_{pk}(i - c_k(i)),\ k \in \{m+1,\dots,n\}$$

in which $c_k(i)$ denotes the discrete time varying delay in transmitting the wave from 'plant-k' to the power junction. Last, the input wave to the controller u_{cj} is a delayed version of the outgoing wave from the *power junction* $u_j,\ j \in \{1,\dots,m\}$ such that

$$u_{cj}(i) = u_j(i - c_j(i)),\ j \in \{1,\dots,m\}$$

in which $c_j(i)$ denotes the discrete time varying delay in transmitting the wave from the power junction to 'controller-j'. In Fig. 1 the delays are represented as fixed for the discrete time case (i.e. z^{-c_j}).

2.2 The Power Junction

The *power junction* Kottenstette et al. (2009)[Definition 1] depicted in Fig. 1 provides a general way to interconnect multiple plants to multiple controllers, and we shall show that it can be implemented in numerous ways.

Assumption 1. n systems are interconnected to a power junction using the corresponding wave variable pairs (u_1, v_1), (u_2, v_2), ..., (u_n, v_n) as indicated in Fig. 1). The *power-output* pairs are denoted (u_j, v_j), $j \in \{1,\dots,m\}$ (in which $u_j \in \mathbb{R}^{m_s}$ is an outgoing wave and $v_j \in \mathbb{R}^{m_s}$ is an incoming wave to the power junction). The *power-input* pairs are denoted (u_k, v_k), $k \in \{m+1,\dots,n\}$ (in which $u_k \in \mathbb{R}^{m_s}$ is an incoming wave and $v_k \in \mathbb{R}^{m_s}$ is an outgoing wave from the power junction).

The power junction is implemented such that (9) holds.

$$\sum_{k=m+1}^{n} (u_k^{\mathsf{T}} u_k - v_k^{\mathsf{T}} v_k) \geq \sum_{j=1}^{m} (u_j^{\mathsf{T}} u_j - v_j^{\mathsf{T}} v_j) \qquad (9)$$

In words, the total *power-input* is always greater than or equal to the total *power-output* from the power junction.

Definition 2. Kottenstette et al. (2009)[Definition 2] An *averaging power junction* as described by Assumption 1 is implemented such that each l^{th} component ($l \in \{1,\dots,m_s\}$) of the outgoing waves (denoted v_{k_l}) are computed from the respective l^{th} component of the incoming waves (denoted v_{j_l}) as follows:

$$\mathsf{sf}_v = \frac{|\sum_{j=1}^{m} v_{j_l}|}{\sum_{j=1}^{m} |v_{j_l}|}$$

$$v_{k_l} = \mathsf{sf}_v \mathtt{sgn}(\sum_{j=1}^{m} v_{j_l}) \frac{\sqrt{\sum_{j=1}^{m} v_{j_l}^2}}{\sqrt{n-m}},\ k \in \{m+1,\dots,n\}$$

$$= \frac{v_{1_l}}{\sqrt{n-1}} \text{ when } m = 1$$

Similarly, each l^{th} component ($l \in \{1, \ldots, m_s\}$) of the outgoing waves (denoted u_{j_l}) are computed from the respective l^{th} component of the incoming waves (denoted u_{k_l}) as follows:

$$\mathsf{sf}_u = \frac{|\sum_{k=m+1}^{n} u_{k_l}|}{\sum_{k=1+1}^{n} |u_{k_l}|}$$

$$u_{j_l} = \mathsf{sf}_u \mathtt{sgn}(\sum_{k=m+1}^{n} u_{k_l}) \frac{\sqrt{\sum_{k=m+1}^{n} u_{k_l}^2}}{\sqrt{m}},\ j \in \{1, \ldots, m\}$$

$$= \mathsf{sf}_u \mathtt{sgn}(\sum_{k=2}^{n} u_{k_l}) \sqrt{\sum_{k=2}^{n} u_{k_l}^2} \text{ when } m = 1.$$

In addition to evaluating the averaging power junction, we introduce the *consensus power junction*.

Definition 3. A *consensus power junction* in which n systems are interconnected as described by Assumption 1 is implemented so that the incoming wave from plant n denoted $u_n(i)$ is related to the outgoing wave to controller 1 denoted $u_1(i)$ as follows

$$u_1(i) = u_n(i). \tag{10}$$

If $m > 1$ then the incoming wave $v_j(i)$ is related to the outgoing wave $u_{j+1}(i)$ to the next controller such that

$$u_{j+1}(i) = v_j(i)\ j \in \{1, \ldots, m-1\}. \tag{11}$$

Next, the final output from the m^{th} controller is connected to the first plant such that

$$v_{m+1}(i) = v_m(i). \tag{12}$$

If $n > m+1$ then the incoming wave $u_k(i)$ is related to the outgoing wave $v_{k+1}(i)$ such that

$$v_{k+1}(i) = u_k(i)\ k \in \{m+1, \ldots, n-1\}. \tag{13}$$

Lemma 4. The *consensus power junction* satisfies (9) as an equality and is therefore a *lossless power junction.*

2.3 Passive Sampling and Holding.

In Kottenstette et al. (2008) it was shown how a passive sampler (PS) a passive hold (PH) in conjunction with a *inner-product equivalent sampler* (*IPES*) and zero-order-hold (*ZOH*) can be used to achieve a L_2^m-stable system consisting of a passive robot and a digital controller. As can be seen in Fig. 1 we have connected the PS and PH to each plant, while connecting the (*IPES*) and zero-order-hold (*ZOH*) block to each digital controller in order to relate $r_{cj}(i)$ to $r_{cj}(t)$ and $e_{cj}(i)$ to $e_{cj}(t)$ in a passivity preserving manner. Therefore we recall the following set of definitions:

Definition 5. The passive samplers denoted (PS_k) and the corresponding passive holds denoted (PH_k) $\forall k \in \{m+1, \ldots, n\}$ must be implemented such that the following inequality is satisfied $\forall N > 0$:

$$\int_0^{NT_s} (u_{pk}^{\mathsf{T}}(t) u_{pk}(t) - v_{pk}^{\mathsf{T}}(t) v_{pk}(t)) dt - \sum_{i=0}^{N-1} (u_{pk}^{\mathsf{T}}(i) u_{pk}(i) - v_{pk}^{\mathsf{T}}(i) v_{pk}(i)) \geq 0. \tag{14}$$

This condition ensures that no energy is generated by the sample and hold devices, and thus, passivity is preserved.

One way to implement the PS and PH is to use the *averaging passive sampler and hold.*

Definition 6. The *averaging passive samplers* denoted (PS_k) and the corresponding *averaging passive holds* denoted (PH_k) $\forall k \in \{m+1, \ldots, n\}$ is implemented such that for each l^{th} component ($l \in \{1, \ldots, m_s\}$) of the discrete-time-sampled waves $u_{pk}(i) \in \mathbb{R}^{m_s}$ (denoted $u_{pk_l}(i)$) is determined from the respective l^{th} component of the continuous-time wave $u_{pk}(t) \in \mathbb{R}^{m_s}$ (denoted $u_{pk_l}(t)$) using PS_k as follows:

$$u_{pk_l}(i) = \sqrt{\int_{(i-1)T_s}^{iT_s} u_{pk_l}^2(t) dt}\ \mathtt{sgn}(\int_{(i-1)T_s}^{iT_s} u_{pk_l}(t) dt) \tag{15}$$

and the continuous-time wave $v_{pk}(t) \in \mathbb{R}^{m_s}$ is determined from the discrete-time waves $v_{pk}(i) \in \mathbb{R}^{m_s}$ in terms of each of their respective l^{th} components using PH_k as follows:

$$v_{pk_l}(t) = \frac{1}{\sqrt{T_s}} v_{pk_l}(i),\ t \in\ [iT_s, (i+1)T_s). \tag{16}$$

Using a PS and PH such as the *averaging passive sampler and hold* we can now relate continuous time variables to discrete time wave variables associated with each plant G_{pk}, $k \in \{m+1, \ldots, n\}$. Substituting (5) into (14) results in the following inequality for each plant

$$\int_0^{NT_s} f_{pk}^{\mathsf{T}}(t) e_{dck}(t) \geq \sum_{i=0}^{N-1} (u_{pk}^{\mathsf{T}}(i) u_{pk}(i) - v_{pk}^{\mathsf{T}}(i) v_{pk}(i)). \tag{17}$$

Next, we would like to determine how (17) relates to the corresponding pair of waves entering and leaving the *power junction* $(u_k(i), v_k(i))$. In order to do so, we recall that (Kottenstette and Chopra, 2009, Proposition 7) (which we shall refer to as Proposition 7) summarizes observations made in Chopra et al. (2008); Stramigioli et al. (2005); Kottenstette and Antsaklis (2007, 2008b)) by stating the necessary time varying delay and data-loss conditons in order for

$$\sum_{i=0}^{N-1} u^{\mathsf{T}}(i) u(i) - v_d^{\mathsf{T}}(i) v_d(i) \geq \sum_{i=0}^{N-1} u_d^{\mathsf{T}}(i) u_d(i) - v^{\mathsf{T}}(i) v(i) \tag{18}$$

to be satisfied for all $N > 0$. Random data dropouts, and fixed delays as well as the TCP/IP transmission protocol will allow (18) to hold, however the UDP protocol could replicate packets and cause (18) to not hold. Applications which choose to use UDP can be easily modified to satisfy Proposition 7-IV by simply not processing duplicate packets.

Corollary 7. All $n-m$ continuous time plant (flows $f_{pk}(t)$ and effort $e_{dck}(t)$) pairs depicted in Fig. 1 are related to their respective pair of waves entering and leaving the *power junction* $(u_k(i), v_k(i))$ such that $\forall k \in \{m+1, \ldots, n\}$

$$\int_0^{NT_s} f_{pk}^{\mathsf{T}}(t) e_{dck}(t) \geq \sum_{i=0}^{N-1} (u_k^{\mathsf{T}}(i) u_k(i) - v_k^{\mathsf{T}}(i) v_k(i))$$

$$\langle f_{pk}(t), e_{dck}(t) \rangle_{NT_s} \geq \| (u_k(i))_N \|_2^2 - \| (v_k(i))_N \|_2^2$$

$$\langle f_{pk}, e_{dck} \rangle_{NT_s} \geq \| (u_k)_N \|_2^2 - \| (v_k)_N \|_2^2 \tag{19}$$

is satisfied if the wave variable communication time-delays $c_k(i) = d_u(i)$, $p_k(i) = d_v(i)$ satisfy any of the conditions listed in Proposition 7.

See (Kottenstette and Chopra, 2009, Appendix A) for an explanation of the short hand notation used in (19), since T_s is typically not an integer, we will typically drop the i or t symbol and use N to refer to extended discrete-time l_2^m norms and NT_s to refer to extended L_2^m norms. In an analogous manner we can relate the control effort and flow variables $(e_{cj}(i), f_{dpj}(i))$ to the power junction wave variables $(u_j(i), v_j(i)\ \forall j \in \{1, \ldots, m\}$ for the m-digital controllers.

Corollary 8. All m discrete time controller (flows $f_{dpj}(i)$ and efforts $e_{cj}(i)$) pairs depicted in Fig. 1 are related to their respective pair of waves leaving and entering the *power junction* $(u_j(i), v_j(i))$ such that $\forall j \in \{1, \ldots, m\}$

$$\begin{aligned}\|(u_j)_N\|_2^2 - \|(v_j)_N\|_2^2 \geq& \|(u_{cj})_N\|_2^2 - \|(v_{cj})_N\|_2^2\\ \|(u_j)_N\|_2^2 - \|(v_j)_N\|_2^2 \geq& \langle e_{cj}, f_{dpj}\rangle_N \end{aligned}\tag{20}$$

is satisfied if the wave variable communication time-delays $c_j(i) = d_u(i)$, $p_j(i) = d_v(i)$ satisfy any of the conditions listed in Proposition 7.

Which leads us to the following lemma.

Lemma 9. The m discrete time controller (flows $f_{dpj}(i)$ and efforts $e_{cj}(i)$) pairs $j \in \{1, \ldots, m\}$ are related to the $n-m$ continuous time plant (flows $f_{pk}(t)$ and effort $e_{dck}(t)$) pairs $k \in \{m+1, \ldots, n\}$ depicted in Fig. 1 as follows

$$\sum_{k=m+1}^{n} \langle f_{pk}(t), e_{dck}\rangle_{NT_s} \geq \sum_{j=1}^{m} \langle e_{cj}(i), f_{dpj}(i)\rangle_N \tag{21}$$

if the wave variable communication time-delays $c_j(i) = c_k(i) = d_u(i)$, $p_j(i) = p_k(i) = d_v(i)$ satisfy any of the conditions listed in Proposition 7.

In order to show L_2^m stability of our digital control network depicted in Fig. 1 we need to relate $\forall j \in \{1, \ldots, m\}$ the discrete-time reference and effort variables associated with each digital controller G_{cj} (denoted by the respective tuple $(r_{cj}(i), e_{cj}(i))$) to a continuous-time reference and effort variable counterpart which we denote by the respective tuple $(r_{cj}(t), e_{cj}(t))$. In order to make this comparison we used the *inner-product equivalent sampler* (denoted IPES_j in Fig. 1) and a zero-order-hold (denoted ZOH_j in Fig. 1). We will refer to the pair of these devices as the *inner-product equivalent sample and hold* (*IPESH*) (Kottenstette and Antsaklis, 2007, Definition 4), Kottenstette et al. (2008).

Definition 10. The *m-inner-product equivalent sample and hold*'s depicted in Fig. 1 by the pair of respective symbols $(\mathsf{IPES}_j, \mathsf{ZOH}_j)$ $j \in \{1, \ldots, m\}$ in which the inputs are denoted by the pair $(r_{cj}(t), e_{cHj}(i))$ and the outputs are denoted by the pair $(r_{cSj}(i), e_{cj}(t))$. The *inner-product equivalent sampler* (*IPES*) is implemented by sampling $r_{cj}(t)$ at a rate (T_s) such that $\forall N > 0$:

$$x(t) = \int_0^t r_{cj}(\tau)d\tau, \quad r_{cSj}(i) = x((i+1)T_s) - x(iT_s). \tag{22}$$

The *ZOH* is implemented as follows:

$$e_{cj}(t) = e_{cHj}(i),\ t \in [iT_s, (i+1)T_s) \tag{23}$$

Corollary 11. Using the *IPESH* as stated in Definition 10 we have that

$$\langle e_{cj}, r_{cj}\rangle_{NT_s} = \langle e_{cHj}, r_{cSj}\rangle_N \text{ holds.} \tag{24}$$

Using the *ZOH* as stated in Definition 10 we also have the property that

$$\|(e_{cj})_{NT_s}\|_2^2 = T_s\|(e_{cHj})_N\|_2^2 \text{ holds.} \tag{25}$$

Finally Fig. 1 possesses some scalar scaling gains $k_s \in \mathbf{R}^+$ to account for the using the power-junction, PS and PH and the *IPESH*, such that for all $j \in \{1, \ldots, m\}$:

$$r_{cj}(i) = -k_{sj}r_{cSj}(i) \quad \text{and } e_{cj}(i) = -\frac{1}{k_{sj}}e_{cHj}(i). \tag{26}$$

Using Corollary 11 and (26) we have the following

$$\langle e_{cj}, r_{cj}\rangle_N = \langle e_{cHj}, r_{cSj}\rangle_N = \langle e_{cj}, r_{cj}\rangle_{NT_s} \tag{27}$$

$$\|(e_{cj})_N\|_2^2 = \frac{1}{k_{sj}^2}\|(e_{cHj})_N\|_2^2 = \frac{1}{T_s k_{sj}^2}\|(e_{cj})_{NT_s}\|_2^2. \tag{28}$$

2.4 L_2^m Stable Power Junction Networks

Fig. 1 depicts m controllers interconnected to $n-m$ plants using a *power junction*. It can be shown that this network will remain L_2^m*-stable* when subject to either fixed delays and/or data dropouts. Furthermore we can show how to safely handle time varying delays by dropping duplicate transmissions from the *power junction*. Refer to (Kottenstette and Chopra, 2009, Appendix A) for corresponding definitions or nomenclature.

Theorem 12. For the network controlled system depicted in Fig. 1, assume all the wave variable communication time-delays $c_j(i) = c_k(i) = d_u(i)$, $p_j(i) = p_k(i) = d_v(i)$ satisfy any one of the conditions listed in Proposition 7. Then the system is:

I. L_2^m*-stable* if all plants $G_{pk}(e_{pk}(t))$, $k \in \{m+1, \ldots, n\}$ and all controllers $G_{cj}(f_{cj}(i)), j \in \{1, \ldots, m\}$ are *strictly-output passive*.

II. *passive* if all plants $G_{pk}(e_{pk}(t))$, $k \in \{m+1, \ldots, n\}$ and all controllers $G_{cj}(f_{cj}(i)), j \in \{1, \ldots, m\}$ are *passive*.

A sketch of the proof is provided, a rigorous proof is in Kottenstette and Chopra (2009). Using Lemma 9 we can show that

$$\begin{aligned}&\sum_{k=m+1}^{n} \langle f_{pk}, r_{pk}\rangle_{NT_s} + \sum_{j=1}^{m} \langle e_{cj}, r_{cj}\rangle_{NT_s} \geq\\ &\epsilon[\sum_{k=m+1}^{n} \|(f_{pk})_{NT_s}\|_2^2 + \sum_{j=1}^{m} \|(e_{cj})_{NT_s}\|_2^2] - \beta\end{aligned} \tag{29}$$

in which $\epsilon = \min(\epsilon_{pk}, \frac{\epsilon_{cj}}{T_s k_s^2})$, $k \in \{m+1, \ldots, n\}$ $j \in \{1, \ldots, m\}$ and $\beta = \sum_{k=m+1}^{n} \beta_{pk} + \sum_{j=1}^{m} \beta_{cj}$. Thus (29) satisfies (Kottenstette and Chopra, 2009, Definition 22-iii) for *strictly-output passivity* in which the input is the row vector of all controller and plant inputs $[r_{c1}, \ldots, r_{cm}, r_{p(m+1)}, \ldots, r_{pn}]$, and the output is the row vector of all controller and plant outputs $[e_{c1}, \ldots, e_{cm}, f_{p(m+1)}, \ldots, f_{pn}]$. When we let $\epsilon_{pk} = \epsilon_{cj} = 0$ we see that all the plants and controllers are *passive*, therefore the system depicted in Fig. 1 is *passive*.

2.5 Steady State Response of Networked Control System

It is desired to relate the controller reference inputs $\{r_{c1}(t), \ldots, r_{cm}(t)\}$ and plant disturbance inputs $\{r_{p(m+1)}(t), \ldots, r_{pn}(t)\}$ to the corresponding controller efforts $\{e_{c1}(t)$

$,\ldots,\ e_{cm}(t)\}$ and plant flows $\{f_{p(m+1)}(t)\ ,\ldots,\ f_{pn}(t)\}$. Since our stability results apply to both linear and nonlinear systems we will focus our initial analysis to the steady-state case $\lim_{t\to\infty} f_{p(m+1)}(t)$. In particular, we determine the steady-state system responses when using either the averaging power junction or the consensus power junction under the following assumptions.

Assumption 13. Each plant, denoted $G_{pk} : e_{pk}(t) \to f_{pk}(t)\ k \in \{m+1,\ldots,n\}$, is a single-input-single-output (SISO) system with a steady-state gain denoted k_{pk} such-that

$$f_{pk}(0) = 0,\ k_{pk} = \lim_{t\to\infty} \frac{f_{pk}(t)}{e_{pk}(t)},\ e_{pk}(t) = \begin{cases} 0, & t < 0 \\ e_{pk}, & t \geq 0. \end{cases}$$

In a similar manner each controller, denoted $G_{cj} : f_{cj}(i) \to e_{cj}(i)\ j \in \{1,\ldots,m\}$, is a SISO system with a steady-state gain denoted k_{cj} such-that

$$e_{cj}(0) = 0,\ k_{cj} = \lim_{i\to\infty} \frac{e_{cj}(i)}{f_{cj}(i)},\ f_{cj}(i) = \begin{cases} 0, & i < 0 \\ f_{cj}, & i \geq 0. \end{cases}$$

Furthermore, for simplicity, all wave variable communication time-delays $c_k(i) = d_u(i) = 1$, $p_k(i) = d_v(i) = 1$. To aid with the steady-state analysis we assume for the PS/PH blocks that

$$u_{pk}(i) = \sqrt{T_s} u_{pk}(iT_s) \text{ and} \tag{30}$$
$$v_{pk}(iT_s) = \frac{v_{pk}(i)}{\sqrt{T_s}}. \tag{31}$$

In addition we assume for the *IPESH* blocks that

$$r_{cSj}(i) = T_s r_{cj}(iT_s) \text{ such that}$$
$$r_{cj}(i) = -k_{sj} T_s r_{cj}(iT_s) \text{ and} \tag{32}$$
$$e_{cj}(iT_s) = e_{cHj}(i) = -k_{sj} e_{cj}(i). \tag{33}$$

Lemma 14. Under the assumptions listed in Assumption 13, the following equations hold for the plants interconnected by the power junction control network depicted in Fig. 1:

$$u_{pk}(i) = \frac{bk_{pk}-1}{bk_{pk}+1} v_{pk}(i) + \frac{\sqrt{T_s 2b} k_{pk}}{bk_{pk}+1} r_{pk}(iT_s) \tag{34}$$
$$f_{pk}(iT_s) = \frac{\sqrt{2b} k_{pk}}{\sqrt{T_s}(bk_{pk}+1)} v_{pk}(i) + \frac{k_{pk}}{bk_{pk}+1} r_{pk}(iT_s). \tag{35}$$

Lemma 15. Under the assumptions listed in Assumption 13, the following equations hold for the controllers inter-connected by the power junction control network depicted in Fig. 1:

$$v_{cj}(i) = \frac{-k_{cj}+b}{k_{cj}+b} u_{cj}(i) + \frac{\sqrt{2b} T_s k_{cj} k_{sj}}{k_{cj}+b} r_{cj}(iT_s) \tag{36}$$
$$e_{cj}(iT_s) = -\frac{\sqrt{2b} k_{cj} k_{sj}}{k_{cj}+b} u_{cj}(i) + \frac{T_s b k_{cj} k_{sj}^2}{k_{cj}+b} r_{cj}(iT_s). \tag{37}$$

Theorem 16. Under the assumptions listed in Assumption 13, the following state equations can be used to determine the steady state response of the power junction control network depicted in Fig. 1 when using the averaging power junction:

$$u_k(i) = \frac{bk_{pk}-1}{bk_{pk}+1} v_{m+1}(i-2) + \frac{\sqrt{T_s 2b} k_{pk}}{bk_{pk}+1} r_{pk}$$
$$v_j(i) = \frac{-k_{cj}+b}{k_{cj}+b} u_1(i-2) + \frac{\sqrt{2b} T_s k_{cj} k_{sj}}{k_{cj}+b} r_{cj}$$
$$u_1(i-1) = \mathsf{sf}_u \mathtt{sgn}\Big(\sum_{k=m+1}^{n} u_k(i-1)\Big) \frac{\sqrt{\sum_{k=m+1}^{n} u_k^2(i-1)}}{\sqrt{m}}$$
$$v_{m+1}(i-1) = \mathsf{sf}_v \mathtt{sgn}\Big(\sum_{j=1}^{m} v_j(i-1)\Big) \frac{\sqrt{\sum_{j=1}^{m} v_j^2(i-1)}}{\sqrt{n-m}}.$$

Likewise the steady-state outputs $f_{pk}(iT_s)\ e_{cj}(iT_s)$ are computed by substituting $v_{pk} = v_{m+1}(i-1)$ into (35) and substituting $u_{cj} = u_1(i-1)$ into (37) respectively.

It is a straight forward exercise for the reader to apply Lemma 14, Lemma 15, Definition 2, and Assumption 13 to verify Theorem 16. For the case of the consensus junction, a closed form solution can be found as stated in Theorem 17.

Theorem 17. Consider the case of a single controller and $n-1$ plants. Under the assumptions listed in Assumption 13, using (34), (35), (36) and (37), employing the consensus power junction, the following steady state equations hold:

$$\begin{aligned} u_{pk}(i) = &\left(\prod_{l=2}^{k} \alpha_l\right) \beta_1 u_{c1}(i) + \left(\prod_{l=2}^{k} \alpha_l\right) \beta_2 r_{c1}(iT_s) \\ &+ \sum_{l=2}^{k} \left(\prod_{s=l}^{k} \gamma_s\right) r_{pl}(iT_s) \end{aligned} \tag{38}$$

where $\alpha_k = \frac{bk_{pk}-1}{bk_{pk}+1}$, $\gamma_k = \frac{\sqrt{T_s 2b} k_{pk}}{bk_{pk}+1}$, $\beta_1 = \frac{-k_{c1}+b}{k_{c1}+b}$ and $\beta_2 = \frac{\sqrt{2b} T_s k_{c1} k_{s1}}{k_{c1}+b}$ and

$$u_{c1}(i) = \frac{\left(\prod_{k=2}^{n} \alpha_k\right) \beta_2 r_{c1}(iT_s) + \sum_{k=2}^{n} \left(\prod_{s=k}^{n} \gamma_s\right) r_{pk}(iT_s)}{1 - \left(\prod_{k=2}^{n} \alpha_k\right) \beta_1} \tag{39}$$

The corresponding outputs $f_{pk}(iT_s)$ can be obtained by additionally using (35).

Furthermore, for the special case of a Proportional-Integral (PI) controller, if $bk_{pk} >> 1$, $k_{s1} = \frac{1}{\sqrt{T_s}}$, and all disturbances are zero, then $f_{pk}(iT_s) = r_{c1}(iT_s)\ \ \forall k$.

3. CONCLUSIONS

A constructive method has been presented which allows the user to construct digital control networks in which passive plants can be interconnected in a manner such that L_2^m-stability is guaranteed. The averaging power junction creates a highly parallel network. Whereas the consensus power junction interconnects waves in a series like manner. The steady-state analysis for the averaging power-junction predicts consensus when the steady-state gains for every plant must be the same, and the controllers steady-state gains must be large. For the consensus power-junction, consensus is only possible when all $bk_{pk} >> 1$ and $k_{cj} >> 1$.

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